Evaluation of methods for phase detection in biosignals

Internship Project Report

submitted by

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Statutory declaration

I declare that I have authored this thesis independently, that I have not used other than the declared sources/resources, and that I have explicitly marked all material which has been quoted either literally or by content from the used sources.

Karlsruhe, July 24, 2017

Abstract

Cardiac diseases are one of the major reasons for death worldwide. The failure in early diagnosis of these diseases reduces the chance of the survival amongst humans. Pre diagnosis of these diseases is essential and carries utmost priority. There are number of parameters which can be evaluated from bio signals helping in pre diagnosis. The instantaneous phase of the signal is an important parameter in biosignal analysis. The instantaneous phase of signals from the electrical impedance tomography is supposed to include, next to the ventilation part, details describing the distribution of perfusion in the torso and therefore in the lungs. This report carries out an analysis to provide the best mathematical model for extraction of the instantaneous phase from bio signals in presence of different type of artefacts. This report also provides the extent by which a particular mathematical model is better than the others and, in turn the reliablity of the method.

Acknowledgments

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Contents

CHAPTER 1

Introduction

Cardiovascular diseases are responsible for the largest number of deaths worldwide. Most heart attacks and strokes could be prevented if some method of pre-monitoring and pre-diagnosing was provided. In particular, early detection of abnormalities in the function of the heart can be valuable for clinicians. Studying the electrocardiogram (ECG) and intracardiac electrogram provides an insight to understand life-threatening cardiac conditions [1]. The electrical activity of the heart is generally sensed by monitoring electrodes placed on the skin surface. The signal can be seen as the sum of sinusoids having their own individual instantaneous amplitudes, instantaneous frequency, energy, instantaneous phase and other parameters. The phase of the signal is an important parameter in biosignal analysis. During a catheter ablation for example, it delivers valuable information about the propagation of the excitation wavefront through the heart. Furthermore, the phase of signals from the electrical impedance tomography is supposed to include, next to the ventilation part, details describing the distribution of perfusion in the torso and therefore in the lungs. This project includes the evaluation of instantaneous phase from various mathematical methods applied on measured signals from the clinics. This report includes findings from a literature review and the implementation of various phase detection algorithms and their evaluation on synthetic signals with respect to different quality indices.

CHAPTER 2

Methods

2.1 Contemporary methods for evaluation of phase from biosignals

The literature review of phase detection methods gave numerous ways and transformations to extract phase from a signal. Each method had a different application area.

The various transformations/methods include:

- Short time Fourier transform / Windowed Fourier transform [2]
- Xiao Method [3]
- Weighted least-squares estimation method [4]
- Hilbert Huang transform [5]
- Wigner transform [6]
- Hilbert transform [7]
- Modified Hilbert transform [2]
- Rantner method [8]
- Sinusoidal recomposition [9]

2.2 Methods used in report

This report draws a comparative study of the experimental results from the various transforms used to extract out phase from biosignals. The biosignals that are taken to study the phase in this report are clinically taken respiratory signals. This report includes these following methods for phase extraction (numbered according to their discussion):

1. Hilbert transform

2. Modified Hilbert transform
3. Short time Fourier transform
4. Rantner method
5. Sinusoidal Recomposition

The first five methods mentioned above are the existing methods. Rantner method cited above does not have a name kept by author. This method is called Rantner method subsequently in this report. The next sections provide an insight into the above listed methods.

2.2.1 Hilbert transform

Hilbert transform is a linear operator (transformation) that maps a function u(t) of a real variable to another real variable function *H(u(t))*. H(u(t)) is the Hilbert transform of the signal u(t). The Hilbert transform of a time domain function is also an another time domain function [7].

2.2.1.1 Analytical representation of signal

The Hilbert transform is widely used in signal processing. The main reason for that is that the Hilbert transform creates an analytic representation of real signals.

Let *f(z)* be an analytic function. If *f(z)* on the real line can be written as

$$f(t) = f(t + 0i) = u(t) + i \cdot \hat{u}(t) \tag{2.1}$$

where u(t) and $\hat{u}(t)$ are real-valued functions and a Hilbert transform pair, then f(t) is said to be an analytic signal. Here u(t) is the input signal and $\hat{u}(t)$ is the Hilbert transform of the input signal

The Hilbert transform of u(t) can be thought of as the convolution of u(t) with the function h(t) = 1/(πt). Hilbert transform of u(t) can be mathematically defined as:

$$\hat{u}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{u(k)}{t-k} dk \tag{2.2}$$

where $\hat{u}(t)$ is Hilbert transform of u(t). H(u(t)) has the effect of shifting the phase of the negative frequency components of u(t) by +90 degrees and the phase of the positive frequency components by -90 degrees. When the Hilbert transform is applied twice, the signal is negated i.e. H(H(u)) = -u. From the definition of the analytic signal given in Eq. 2.1, the instantaneous phase of a signal u(t) is the inverse tangent of the ratio of the imaginary part of Hilbert transform to its real part.

$$\psi(t) = tan^{-1}(imag(\hat{u}(t))/real(\hat{u}(t))) \tag{2.3}$$

"The function atan2 computes the principal value of the argument function applied to the complex number x+iy. That is, atan2(y, x) = Arg(x+iy). The argument can be changed by 2π (corresponding to a complete turn around the origin) without making any difference to the angle, but to define atan2 uniquely one uses the principal value in the range $(\pi, \pi]$. That is, $-\pi <$ atan2(y, x) $\leq \pi$" [10] .

2.2.1.2 Algorithm

Algorithm 2.1 Extracting phase out of signal from Hilbert transform. x(n) denotes the input signal. ψ is the phase of the signal

$z(n) \leftarrow hilbert\_transform(x(n))$
$\psi \leftarrow atan2(z(n), x(n))$

2.2.2 Modified Hilbert transform

2.2.2.1 Theory

In the modified Hilbert transform method, the analytical signal is preprocessed in contrast to a different manner than the conventional method. The imaginary part of the analytic signal includes the product of a window function and the impulse response of the Hilbert filter. This product is convolved upto a total length of the window function. This length (L) is chosen according to the frequency to be investigated present in the input signal. In this study, (periodic) Hamming window is used. The impulse response h(m), of the filter is given in Eq. 2.4. The implemented method is taken from [2].

$$h(m) = \begin{cases} (2/\pi m)sin^2(\pi m/2), & \text{for } m \neq 0 \\ 0 & \text{for } m = 0 \end{cases} \tag{2.4}$$

2.2.2.2 Algorithm

Algorithm 2.2 Extracting phase out of signal from Modified Hilbert transform. x(n) denotes the input signal in discrete domain. fc denotes frequency of interest in input signal.

$M \leftarrow sampling\_rate/fc$
$L \leftarrow 2M+1$
$w(m) \leftarrow hamming(L)$
$h(m) \leftarrow according\_to\_Eq.2.4$
$p \leftarrow i \cdot (h(m) \cdot w(m))$
$sum_{conv} \leftarrow convolution_{same}(x, p)$
$y \leftarrow x + sum_{conv}$
$\psi \leftarrow atan2(imag(y), real(y))$

2.2.3 Short time Fourier transform

2.2.3.1 Theory

The Short time Fourier transform (STFT) is often used when it is desirable to have better time localization. The idea is to use a window function w(n) that zeroes all but a chosen time interval. All events in the Fourier transform are then localized to that interval. The windows are shaped so that when they are overlapped (shifted by S samples and summed), their sum $\sum_n w(m-nS)$ is constant for all **m**. This is shown in Figure 2.1 where the windows are overlapped [11].

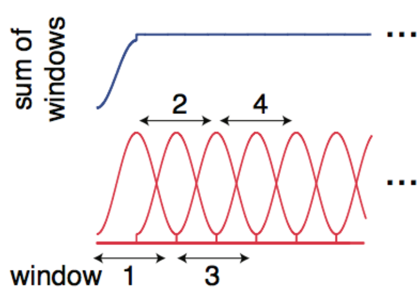

Figure 2.1: A set of overlapping windows is used to zero all but a short segment of the signal. The Fourier transform can then be applied to that segment in order to localize events in time. An overlap factor of 2 is shown [11].

In discrete time, STFT is defined as:

$$STFTx(n)(m,\omega) = \Sigma_{-\infty}^{\infty} x(n)w(n-m)e^{-j\omega n} \quad (2.5)$$

STFT of x(n) is a function of both m (m specifies where the window is nonzero) and frequency ω. Thus, the STFT provides a series of spectral snapshots that move through time. The operation of a STFT-based signal processor is shown in Fig. 2.2. The signal is partitioned into segments by the windows. The FFT is applied to each segment separately (only one processing path is shown). When summed together, the segments reconstruct the original signal. STFT is invertible: it is possible to break the signal into spectral snapshots and then reconstruct the original signal from the snapshots [11].

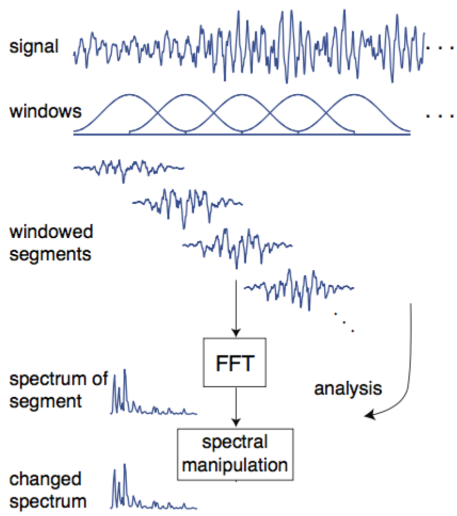

Figure 2.2: A short-time Fourier transform (STFT) is an analysis/synthesis method that begins by windowing a signal into short segments. The FFT is applied to each segment separately and the resulting spectral snapshot can be manipulated in a variety of ways. Image is a modified version of image taken from [11].

2.2.3.2 Approach

In the evaluated STFT approach [2] (which is different from original STFT), a window function (w(m)) is modulated according to the real and imaginary parts of kernel of the STFT, $e^{-j\omega t}$. According to Euler's theorem :

$$e^{-j\omega t} = cos(\omega t) - j \cdot sin(\omega t) \tag{2.6}$$

STFT uses two analysing wavelets namely, sine window function ($w_s(m)$) and cosine window function ($w_c(m)$) which are the product of real and imaginary parts of the STFT kernel with the window function respectively. In this study, (periodic) Hamming window is used. The length of the analysing window is chosen according to the frequency of interest present in the input signal. Multiple convolutions are carried out to extract the phase from the signal. Algorithm 2.3 illustrates the procedure to carry out phase extraction with STFT.

2.2.3.3 Algorithm

Algorithm 2.3 Instantaneous phase of signal from STFT. x(n) denotes the input signal in discrete domain. fc denotes frequency of interest in input signal

$M \leftarrow sampling\_rate/f_c$
$L \leftarrow 2M+1$
$w(m) \leftarrow hamming(L)$
for all $m = -M \longrightarrow M$ **do**
  $w_s(m) \leftarrow w(m) \cdot sin(2\pi(f_c/rate) \cdot m)$
  $w_c(m) \leftarrow w(m) \cdot cos(2\pi(f_c/rate) \cdot m)$
end for
$xs_{conv} \leftarrow convolution_{same}(x, w_s(m))$
$xc_{conv} \leftarrow convolution_{same}(x, w_c(m))$
$\psi \leftarrow arctan2(xs_{conv}, xc_{conv})$

2.2.4 Rantner method

2.2.4.1 Theory

The phase was computed using two consecutive trans-membrane voltages (TMV) distributions in [12]. The phase from this method in this report is calculated by:

$$\psi(x,y,t) = atan2[V_m(x,y,t+\tau) - V_m^*, V_m(x,y,t) - V_m^*], \tag{2.7}$$

with τ being the time increment between two signals. The second signal is just a delayed version of the input (first) signal. If the input signal is a simple periodic sinusoidal wave, V_m^* = 0 can serve the purpose. In algorithm, 2.4 τ is taken equal to 1/4 times the period of the input signal. When there is an offset present in the signal, V_m^* cannot be equal to zero.

2.2.4.2 Algorithm

Algorithm 2.4 Extracting phase out of signal from Rantner method. x(t) denotes the input signal. f_c denotes frequency of interest in input signal

$\tau \leftarrow 1/(4 \cdot fc)$
$V_m^* \leftarrow 0$
$y \leftarrow x(t+\tau) - Vm^*$
$v \leftarrow x(t) - V_m^*$
$\psi \leftarrow arctan2(y, v)$

2.2.5 Sinusoidal Recomposition

2.2.5.1 Theory

In order to perform a robust phase reconstruction, the authors in [9] propose a signal transformation based on the recomposition of the electrogram from sinusoidal wavelets with amplitudes proportional to the negative slope of the electrogram. Application of the sinusoidal recomposition transformation prior to application of the Hilbert transform alleviates effect of some confounding features (eg. offset) on reconstructed phase.
In this method, instantaneous phase is defined as Eq. 2.8

$$\phi(t) = arctan(\frac{-(v(t) - v^*)}{H(v((t)) - v^*}) \tag{2.8}$$

where v(t) is the recomposed signal (sum of sinusoidal wavelets) derived from the input signal and v^* sets the origin of the phase plane with respect to which phase is computed. (H(v(t)) is Hilbert transform of v(t)).

2.2.5.2 Algorithm

The authors proposed the following transformation [13]:

1. The transformed signal is a sum of sinusoidal waves of one period length (called sinusoidal wavelets below).
2. For each time point of the original signal, one sinusoidal wavelet is created.
3. The amplitude of the sinusoidal wavelet is proportional to the slope of the signal at a given time point.
4. A wavelet is generated only if a derivative of the signal is negative.
5. The period of the sinusoidal wavelet is equal to the mean cycle length of the electrogram derived from dominant frequency of given electrogram.

The phase of the recomposed signal was calculated using Eq. 2.8. Since the recomposed signal is a sum of sinusoidal wavelets with a mean value equal to zero and wavelets of the greatest amplitude are clustered around the negative slope of the local deflection, the resultant recomposed signal also has a sinusoidal morphology oscillating around zero value. Based on this consideration, we set v^* (origin of the phase space with respect to which phase is computed) to zero.

2.3 Noise attributes

"In signal processing, noise is a general term for unwanted modifications that a signal may suffer during capture, storage, transmission, processing, or conversion" [14]. A fundamental problem in signal measurement is distinguishing the true underlying signal from the noise.

The characteristics of the signal deviates from the ideal behaviour in presence of noise. Thus, the results are prone to error. The noise may come via instrument, external environment, internal environment setup or human interference itself [15]. Noise introduces distortions in the results and therefore need to be obviated for the correct estimation of the signal and the corresponding phase. This project report takes various noises into consideration that can be present in the signal of interest introduced via different sources. The impact of the noise on the phase of the signal is then studied and a comparative analysis is done among the various transformations discussed above, to get the best possible results out of a noisy signal.
The quality of a signal is often expressed quantitatively as the signal-to-noise ratio (S/N ratio), the ratio of the energy of the signal to the energy of the noise. SNR is used to quantify noise in some of the parameters discussed subsequently in this report.
The following noise attributes are taken into consideration and their results are analyzed:

1. White Gaussian Noise
2. Offset
3. Window Length Variation
4. Phase Shift in Rantner method

2.3.1 White Gaussian Noise

By definition, the random process X(t) is called white noise if $S_X(f)$ (PSD of X(t)) is constant for all frequencies. By convention, the constant is usually denoted by $N_0/2$ [16]

$$S_X(f) = \frac{N_0}{2}, \quad \text{for all } f. \tag{2.9}$$

$$E\left[X(t)^2\right] = \int_{-\infty}^{\infty} S_X(f)\, df = \int_{-\infty}^{\infty} \frac{N_0}{2}\, df = \infty. \tag{2.10}$$

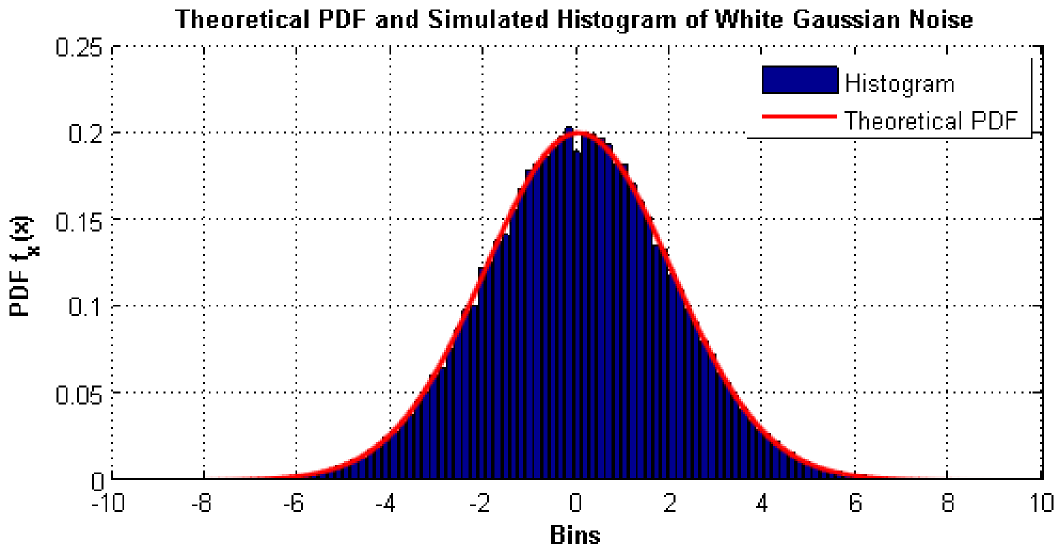

Figure 2.3: Probability density function of white Gaussian noise [17]

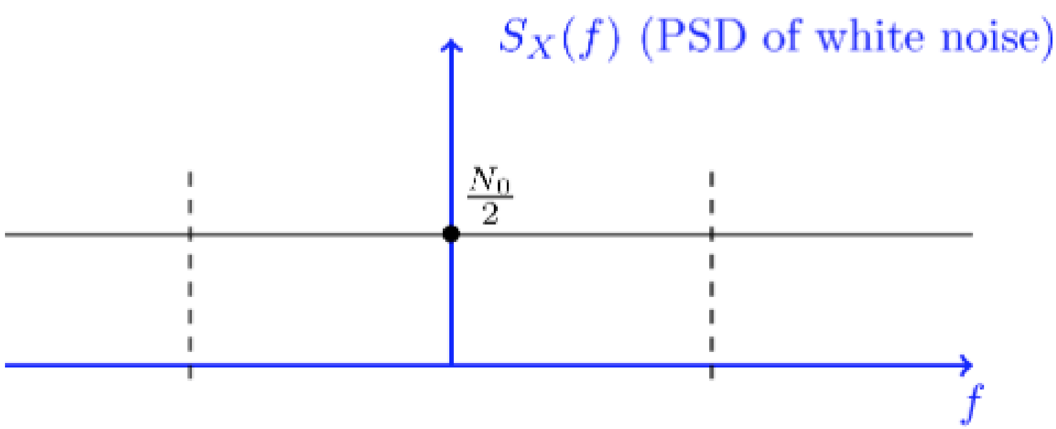

Figure 2.4: Power spectral density of white Gaussian noise [17]

Eq. 2.10 gives the energy of the white Gaussian noise. Thus, white noise, as defined above, has infinite power. In reality, white noise is in fact an approximation to the noise that is observed in real systems. Probability density function $f_x(x)$ of white Gaussian noise is given by Eq. 2.11.

$$f_x(x) = (1/\sqrt{2\pi})e^{-(x-\mu)^2/2\sigma^2} \tag{2.11}$$

where μ is the expected value and σ is the standard deviation in Eq. 2.11. Probability density function and PSD of white Gaussian noise are shown in Fig. 2.3 and Fig. 2.4 respectively. In this study, the expected value μ was chosen to zero. White Gaussian noise of a SNR of -6dB, -3dB, +0dB, +3dB, +6dB, +10dB and +20dB are studied and results are mentioned in section 3 of the report.

2.3.2 Offset

If offset remains as non-zero due to incapable instruments, it acts as a source of noise for the signal and the obtained phase is not the exact reproduction of the original one. The effect of offset on the phase of the signal was studied and results are obtained and mentioned in section 3. Offset of different SNR (-10 dB, -6 dB, -3 dB, 0 dB, +3 dB) were taken and their effects were studied on the phase of signal.

2.3.3 Varying window length

"In signal processing, a window function is a mathematical function that is zero-valued outside of some chosen interval. When another function is multiplied by a window function, the product is also zero-valued outside the interval: all that is left is the part where they overlap. In this study, Hamming window is chosen. The window is optimized to minimize the maximum (nearest) side lobe, giving it a height of about one-fifth that of the Hann window. The window function is given by Eq. 2.12 where $\alpha = 0.54$ and $\beta = 1 - \alpha$ " [18].

$$w(n) = \alpha - \beta cos(\frac{2\pi n}{N-1}) \tag{2.12}$$

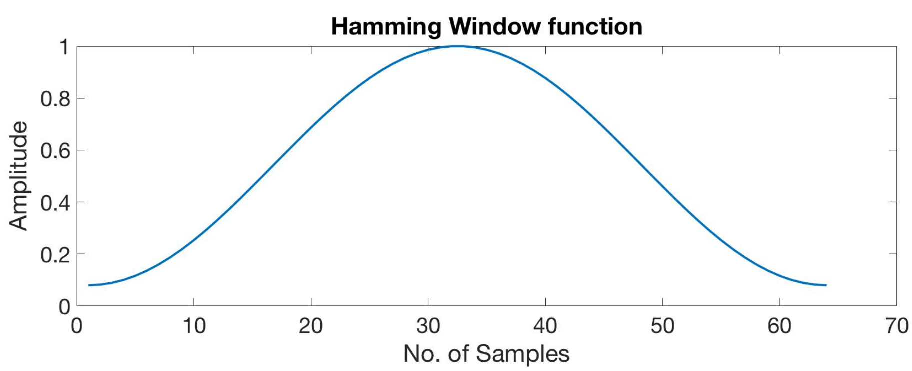

Figure 2.5: Hamming Window function [18]

The window length is an important parameter in signal processing of these signals. A particular window length may or may not serve as a noise for phase extraction of a particular frequency of interest. In the noise free case, the window length (L) was chosen to be equal to L (L=2M+1), where M is Sampling rate / frequency of the interest in input signal. Experimentally, the window length (L = 2M +1) gave best results for the noise free environment.

2.3.4 Phase shift in Rantner method

The instantaneous phase of the signal is extracted from the original signal and its delayed version. The delay in this report is taken in terms of angle. One time period of the input signal corresponds to 2π angle. The delay taken for calculating the phase under ideal conditions by this method was $\pi/2$ i.e. $\frac{1}{4}$ of the period of the input signal. Effective delay(s) were varied to study its impact on phase of the signal. Delays of $\pi/4$ (T/8), π (T/2) and 2π (T) were taken into account in this study.

2.3.5 Wrong estimation of input signal frequency

The frequency of the input signal (sine wave) is 1 kHz. This frequency is required for the phase estimation in Short time Fourier transform method and sinusoidal recomposition method. The frequency of the signal can be estimated incorrectly due to observational errors. This incorrect estimation leads to erroneous results. The effect of wrong estimation of frequency of interest in input signal is studied in 3.

2.3.6 Phase discontinuity

The real world signals are not the ideal complex sinusoidal signals. Irregular phase jumps can be observed in the real world signals. Therefore, evaluating the best method for extracting the phase from a signal having numerous phase discontinuities becomes mandatory. The phase discontinuity was added to the signal between 4th and 5th second. This has been done randomly 100 times. These are then passed as input signals to the various methods. The performance indexes of the phase obtained from the various methods have been mentioned in table 4.16.

2.4 Result parameters

The results in section 3 can be interpreted from the following parameters:

1. Correlation coefficient
2. Mean error
3. Standard deviation of error

2.4.1 Correlation coefficient

The correlation coefficient is a measure that determines the degree to which two variable's movements are associated. The range of values for the correlation coefficient is -1.0 to

1.0. A value of exactly 1.0 means there is a perfect positive relationship between the two variables. For a positive increase in one variable, there is also a positive increase in the second variable. A value of exactly -1.0 means there is a perfect negative relationship between the two variables. This shows the variables move in opposite directions; for a positive increase in one variable, there is a decrease in the second variable. If the correlation is 0, this simply means there is no relationship between the two variables. The strength of the relationship varies in degree based on the value of the correlation coefficient. For example, a value of 0.2 indicates there is a positive relationship between the two variables, but it is weak [19]. Covariance of two variables X and Y is given by:

$$\mathrm{covar}(X,Y) = \mathrm{E}\left[(X - \mathrm{E}[X])(Y - \mathrm{E}[Y])\right], \tag{2.13}$$

where E[X] is the expected value of X, also known as the mean of X [20]. The correlation coefficient of two variables A and B is given by Eq. 2.14.

$$\rho(A,B) = \frac{covar(A,B)}{\sigma_a \sigma_b} \tag{2.14}$$

where σ_a and σ_b are standard deviation of A and B respectively. Correlation coefficient of same variable is equal to 1. $\rho(A,A) = 1$ [21] .

2.4.2 Mean error

The error, in this report, is the difference of the observed phase under the noisy conditions with the phase obtained in noise free condition. The average value of this error for the entire period of observation is called mean error. Larger the mean error, higher is deviation from actual phase obtained. Eq. 2.15 and Eq. 2.16 gives the error and mean error respectively.

$$err = \psi - \phi \tag{2.15}$$

where ψ is the phase obtained under noisy condition and ϕ is the phase obtained under noise free condition.

$$\mu = \frac{\Sigma_T(err)}{T} \tag{2.16}$$

T is the time period of observation.

2.4.3 Standard deviation of error

"Standard deviation is a measure that is used to quantify the amount of variation or dispersion of a set of data values". A low standard deviation indicates that the data points tend to be close to the mean of the set, while a high standard deviation indicates that the data points are spread out over a wider range of values [22]. Eq. 2.17 gives standard deviation of set of N values.

$$\sigma = \sqrt{\frac{\Sigma_N (x_i - \mu)^2}{N}} \tag{2.17}$$

where μ is mean of the data.

CHAPTER 3

Results

3.1 Conditions

3.1.1 Ideal conditions with no type of noise added

No white Gaussian noise was added in the ideal case. The offset of the signal was zero. The window function is equal to 2·(sampling rate/f_c)+1 (f_c = frequency of sine wave).

3.1.2 Effect of addition of white Gaussian noise

White gaussian noise having an expected value equal to zero with varying SNR was taken as input signal and its impact on the phase of the signal was studied. Various SNR levels of the signal were, -6 dB, -3 dB, 0 dB, +3 dB, +10 dB and +20 dB. For analysis, the signal was considered from t = 0 s to t = 10 s.

3.1.3 Effect of addition of offset

Various offsets were added to study the impact of offset on phase extraction methods. The values for the offsets included offset value= -2, offset value= -0.5, and offset of SNR values -10 dB, -6 dB, -3 dB, 0 dB and +3 dB. For analysis, the signal was considered from t = 0 s to t = 10 s.

3.1.4 Effect of change in window length

The effect on phase estimation due to window length is discussed in section 2.12. By varying the size of the window in Modified Hilbert transform method and Short time Fourier transform method, the phase estimation techniques were compared. Hilbert transform and Rantner method do not consider a window length while estimating phase of a signal. Therefore, these two methods were not taken into consideration. The input signal remains undistorted for the evaluation of the window length, as shown in Fig. 3.1. The factors taken into account to modify window length in this study were 0.25, 0.5, 2 and 4.

3.1.5 Effect of phase shift on Rantner method

Rantner method estimates the phase of a signal by delaying it by a definite number of samples. This delay is 1/4 of the period of the signal i.e. $\pi/2$ in the noise free case. The signal was delayed by the phase of $\pi/4$ (1/8 of period of signal), π (half a period) and 2π (one period) and results were illustrated.

3.1.6 Effect of wrong estimation of frequency in the input signal

In real world analysis of bio signals, the exact estimation of frequency of the input signal (of interest) is a challenging task. The incorrect estimation of input signal frequency acts as noise in phase extraction in STFT and Sinusoidal recomposition methods. In this report, frequency was varied in terms of number of samples. For an input signal of 1 Hz and sampling rate of 1 kHz, there are 1000 samples in 1 s. The number of samples were varied from 500 to 2000 in steps of 100 and its effect was studied on phase of the signal.

3.2 Input signal

3.2.1 Ideal conditions with no type of noise added

The input signal was a sine wave having frequency f_c = 1 Hz. Sampling rate was 1 kHz. The signal is plotted on time axis in Fig. 3.1

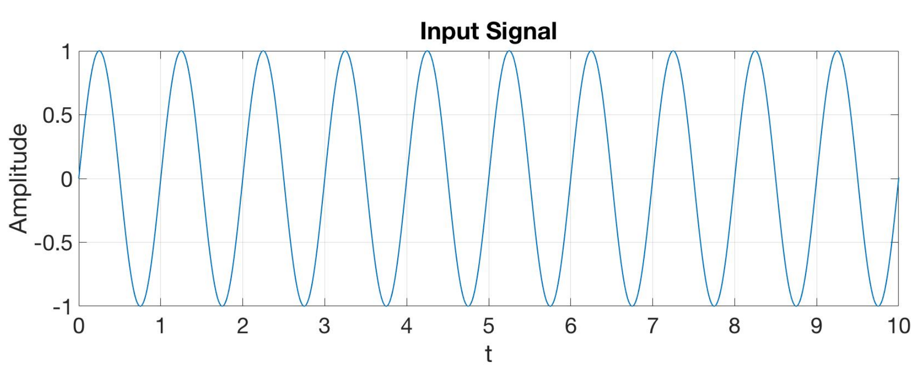

Figure 3.1: Input signal has a frequency of 1 Hz. x=sin($2\pi f_c$t), where t is in seconds.

3.2.2 Effect of addition of white Gaussian noise

The input signal with added white Gaussain noise with SNR -6 dB and +20 dB are shown in Fig. 3.2 and Fig. 3.3. The signal improves in quality as SNR value is increased towards positive side. Figures having a SNR of -3 dB, 0 dB, +3 dB and +10 dB are not plotted as they do not deliver any further information.
The signal with SNR = -6 dB follows the ideal signal but have random jumps at every point. On the other hand, signal having SNR = +20 dB resembles with ideal input signal and is clearly less distorted.

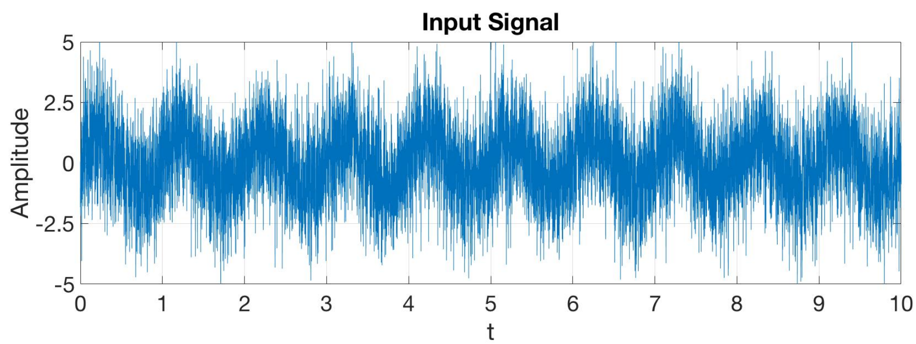

Figure 3.2: Input signal of a SNR -6 dB. t in s.

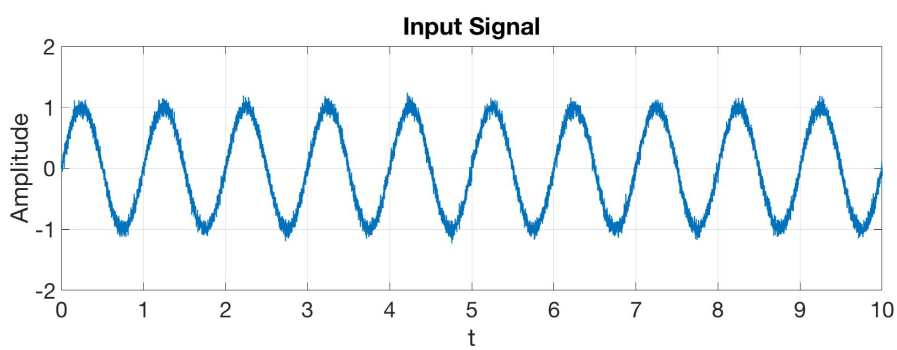

Figure 3.3: Input signal of a SNR +20 dB. t in s.

3.2.3 Effect of addition of offset

The signal having an offset equal to -2 is shown in Fig. 3.4. Signals are shifted by the offset values on the amplitude axis.

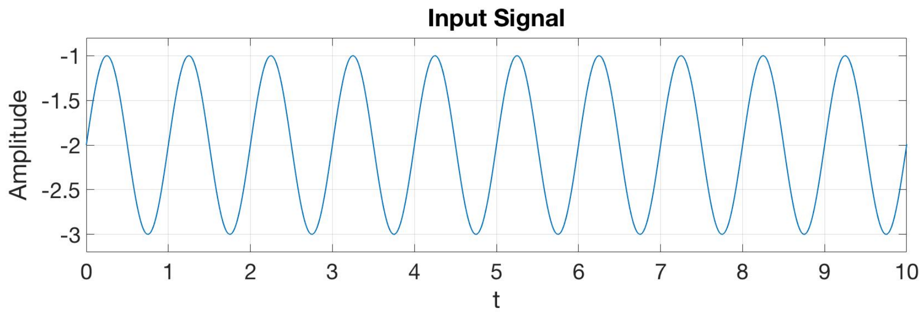

Figure 3.4: Input signal with offset = -2. The signal starts from -2 at t=0. t in s.

3.2.4 Effect of change in window length

The input signal remains undistorted for the evaluation of the window length, as shown in Fig. 3.1.

3.2.5 Effect of phase shift on Rantner method

The input signal remains undistorted for the evaluation of the phase by Rantner method, as shown in Fig. 3.1.

3.2.6 Effect of phase discontinuity

The input signal has a single phase discontinuity between 4th sec and 5th sec. The discontinuity has been generated by a random function generator. The results mentioned in table 4.16 have been calculated by taking the average of the performance parameters for 100 such signals with phase discontinuity present at a different time instant between 4th and 5th second each time. One of the input signal has been shown in Fig. 3.5.

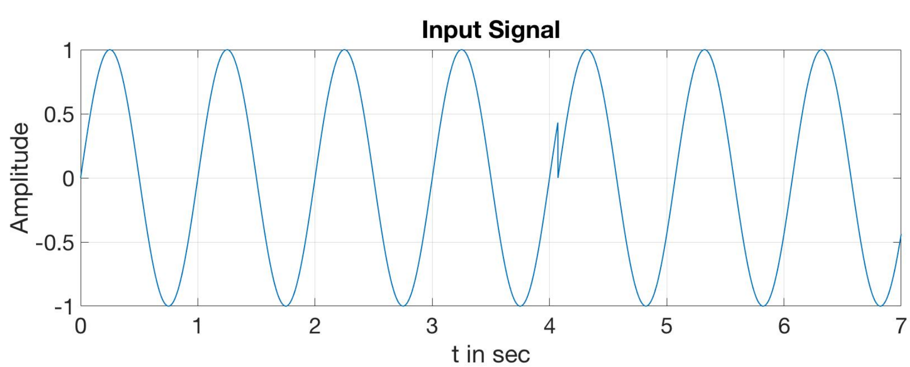

Figure 3.5: Input signal having a phase discontinuity between 4th and 5th second

3.3 Hilbert transform

3.3.1 Ideal conditions with no type of noise added

In subsequent sections, ideal phase refers to the phase in the absence of noise for the corresponding method.

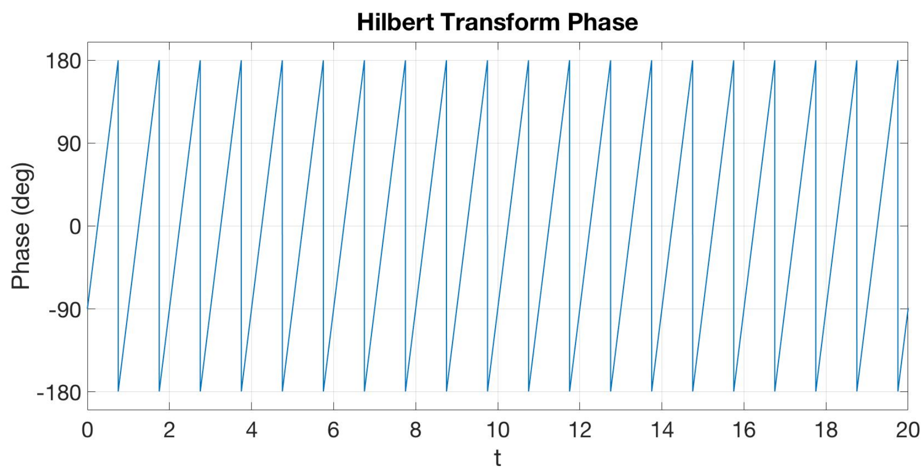

Figure 3.6: Phase estimation from standard Hilbert transform

Since it is Hilbert transform, (as discussed in sec. 3.3.1) the phase starts from (-π/2) at t=0 sec. The first maximum peak comes at t=0.75 sec corresponding to π. The next peak comes at t=1.75 sec corresponding to π. The cycle repeats itself and is therefore periodic with a time period of 1 sec. This transform provided accurate results with no sporadic or eccentric behaviour. The phase at starting point was well defined.

Description : The phase by Hilbert transform is obtained as ratio of the inverse tangent of the imaginary part of the analytic signal to its real part. The imaginary part of the analytic signal is the Hilbert transform of the real part of the analytic signal. As discussed in 2.2.1.2, Hilbert transform adds up a phase of -90 degrees to the positive frequency and +90 degrees to the negative frequencies. Since the input signal is a pure sine wave, Hilbert transform of the input is the negative of cosine of the same signal and, the phase is calculated by Eq. 2.3 ; arctan(- cot x)= x - $\pi/2$ (for x < π). Therefore the phase starts from -$\pi/2$ having a slope of 1. At t = 0.75 sec, after touching π, the phase again starts from - π and increasing with same slope = 1 afterwards. The cycle repeats itself after every second.

3.3.2 Effect of addition of white Gaussian noise

3.3.2.1 Signal with SNR = -6 dB

The phase obtained from Hilbert transform is shown in Fig. 3.7. The phase starts from -104.5 degrees at t = 0. The result is sporadic and fluctuate at a very fast rate and thus is not at all reliable and reproducible. The waveform still lightly follows ideal phase with an approximate time period of 1 s. The phase varies from -179.4 degrees to +179.6 degrees.

3.3.2.2 Signal with SNR = +20 dB

The phase obtained from Hilbert transform is shown in Fig. 3.8. The phase starts from -89.89 degrees at t=0. The waveform follows ideal phase with a time period of 1 sec. The phase varies from -180 degrees to +180 degrees. Although we have a very high SNR, the distortions are still visible.

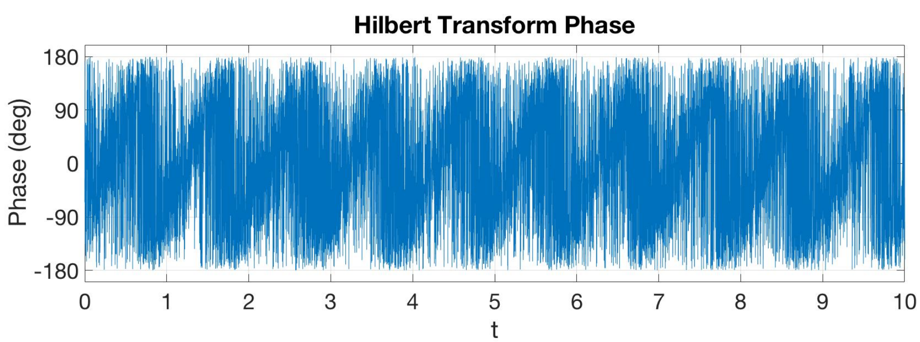

Figure 3.7: Hilbert transform phase from signal of SNR -6dB

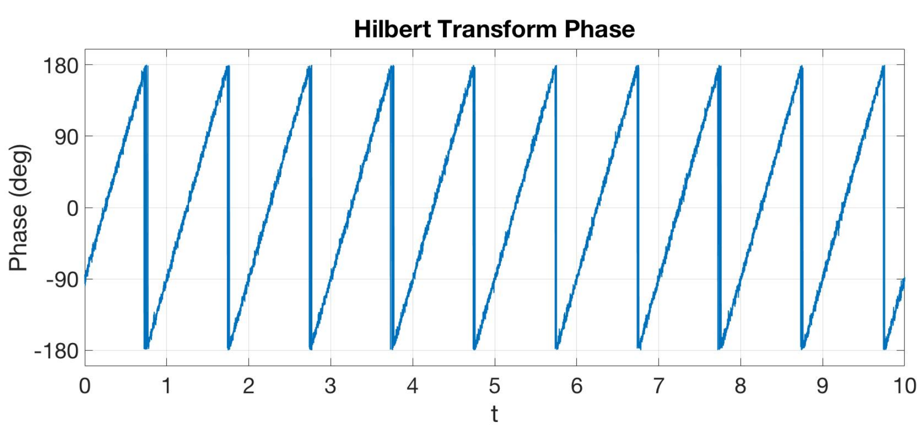

Figure 3.8: Hilbert transform phase of from signal of SNR +20 dB

Table 3.1: Various parameters of phase plot obtained when white Gaussian noise was added (value in degrees) via Hilbert transform

	SNR value (in dB)					
	-6	*-3*	*0*	*+3*	*+10*	*+20*
Starting phase (t=0)	-104.5	-110.9	-104.3	-88.35	-84.8	-89.8
Maximum phase	179.6	179.8	179.9	180	180	180
Minimum phase	-179.4	-179.7	-179.8	-179.8	-179.9	-180

Other plots behave in same manner as above with the signal becoming more accurate with increasing SNR value towards positive side. All signals were periodic having an approximate time period of 1 sec and follows the ideal phase in an irregular manner. The starting phase, the maximum and minimum phase have been given in Table 3.1.

Description

Addition of white Gaussian noise with expected mean equal to zero introduces error in phase calculated from Hilbert transform. Hilbert kernel $\frac{1}{t-k}$ (t is domain in which transform is done and k is the variable with which Hilbert transform is calculated) is multiplied with the input signal which contains noise and is integrated over the whole range of observation. The integration of the noise component does not give value equal to zero and thus, the phase is not well defined, rather jumping instantaneously at every point. Thus, Hilbert transform phase in presence of white Gaussian noise is not reliable and vulnerable to fluctuations. The phase moves closer to the ideal phase as the SNR value of signal is increased towards positive SNR values.

3.3.3 Effect of addition of offset

3.3.3.1 Signal with offset = -2

The phase obtained from Hilbert transform is shown in Fig. 3.9. The results depict Hilbert transform cannot be used with signals having offset value. The phase starts from -153.7 degrees at t=0. The results are periodic having jumps in between and does not reproduce ideal phase. The results are straight lines with small variations at regular time intervals. The phase varies from -179.9 degrees to +180 degrees.

3.3.3.2 Signal with offset = -0.5

The phase obtained from Hilbert transform is shown in Fig. 3.10. The phase starts from -110.5 degrees at t=0. The first maxima occurs at t=0.75 sec. The results is cyclic with a time period of 1 sec. The phase varies from -180 degrees to +180 degrees.

3.3.3.3 Signal with offset = -10 dB

The phase obtained from Hilbert transform is shown in Fig. 3.11. The results are erratic and does not reproduce actual phase. The first maxima occurs at t=0.75 sec. The results does not follow actual phase but is cyclic with a time period of 1 sec. The phase varies from -24.7 degrees to +24.7 degrees.

3.3.3.4 Signal with offset = +3 dB

The phase obtained from Hilbert transform is shown in Fig. 3.12. The phase starts from -66.8 degrees at t=0 and gives better result than in previous case. The first maxima occurs at t=0.75 sec. The phase varies from -180 degrees to +180 degrees.

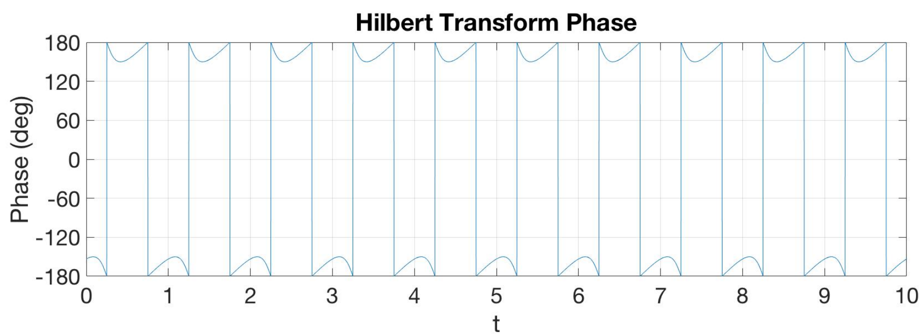

Figure 3.9: Hilbert transform phase with signal having offset = -2

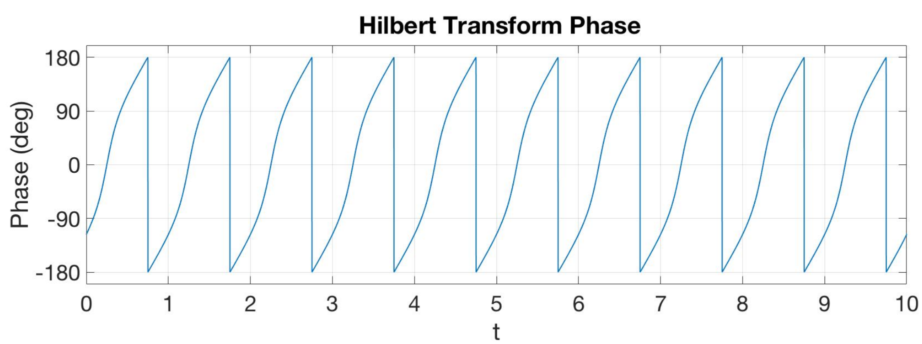

Figure 3.10: Hilbert transform phase with signal having offset = -0.5

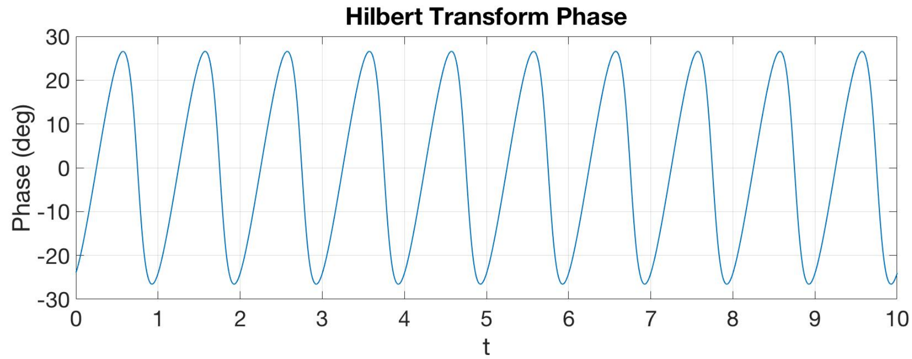

Figure 3.11: Hilbert transform phase with signal having offset = -10dB

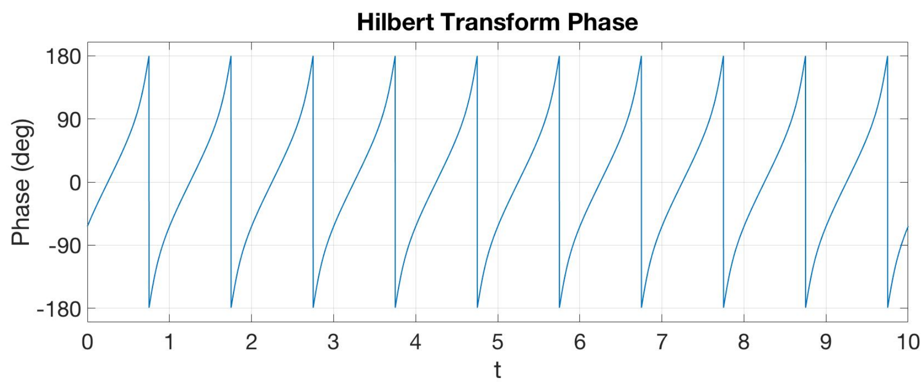

Figure 3.12: Hilbert transform phase with signal having offset = + 3dB

Description

In noise free analysis, the plot of the Hilbert transform of a signal vs the signal is a circle centered about origin having radius equal to amplitude of the signal. Adding offset as a noise shifts the circle in this XY plane. Adding a negative offset, shifts the circle center towards negative axis but having no ordinate. The phase is the argument (taken from X axis) of a point on this circle w.r.t to origin. Adding a large negative offset to the signal as in offset = -2 with regard to the amplitude of the sine wave, phase just jumps between two values, 180 degrees and -180 degrees as the circle appears as a point wrt origin. This point appears to be on X axis and therefore, the argument is either 180 degrees or -180 degrees as shown in Fig. 3.9. There are small fluctuations in the result that indicate the circle appears as very small (not exactly as a point). When the offset is large on a positive side, circle shifts towards positive X axis. Fig. 3.11 shows phase obtained when offset is equal to -10 dB. The argument varies from -24 degrees to +24 degrees and the phase is restricted between these two values. When SNR is increased towards more positive side by decreasing the offset, the result starts looking like the ideal phase.

3.3.4 Effect of phase discontinuity

The phase obtained by Hilbert transform from the input signal having a single phase discontinuity (input signal is discussed in 3.2.6) is shown in Fig.3.13.

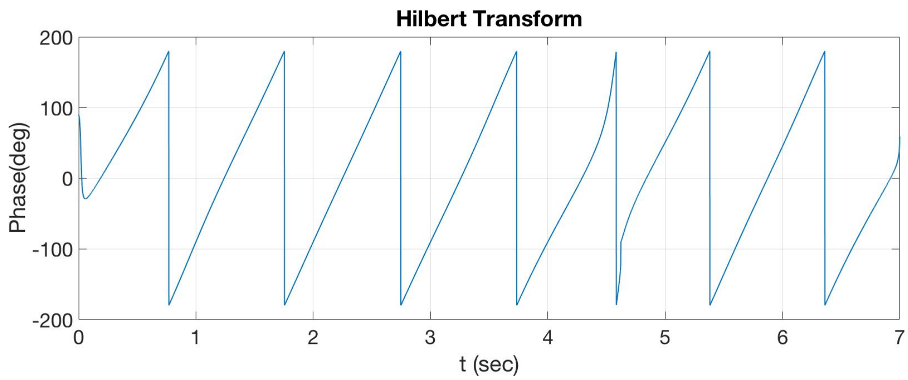

Figure 3.13: Phase estimation from standard Hilbert transform

3.4 Modified Hilbert transform

3.4.1 Ideal conditions with no type of noise added

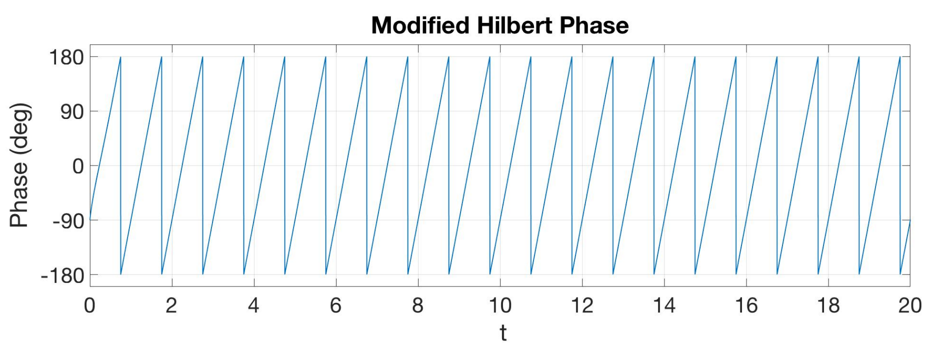

Figure 3.14: Modified Hilbert phase

The phase starts from (-π/2) at t=0 sec. The first maximum peak comes at t=0.75 sec corresponding to π. The next peak comes at t=1.75 sec corresponding to π. The cycle repeats itself and is therefore periodic with a time period of 1 sec. This transform provides accurate results with no sporadic or eccentric behaviour. The phase at starting point is well defined.

3.4.2 Effect of addition of white Gaussian noise

3.4.2.1 Signal with SNR = -6 dB

The phase obtained from Modified Hilbert transform is shown in Fig. 3.15. The phase starts from -32.1 degrees at t=0. The results are sporadic and are not at all reliable and reproducible. The waveform has an approximate time period of 1 sec. The phase varies from -179.3 degrees to +179.9 degrees.

3.4.2.2 Signal with SNR = +20 dB

The phase obtained from Modified Hilbert transform is shown in Fig. 3.16. The waveform still has small fluctuations in the phase.

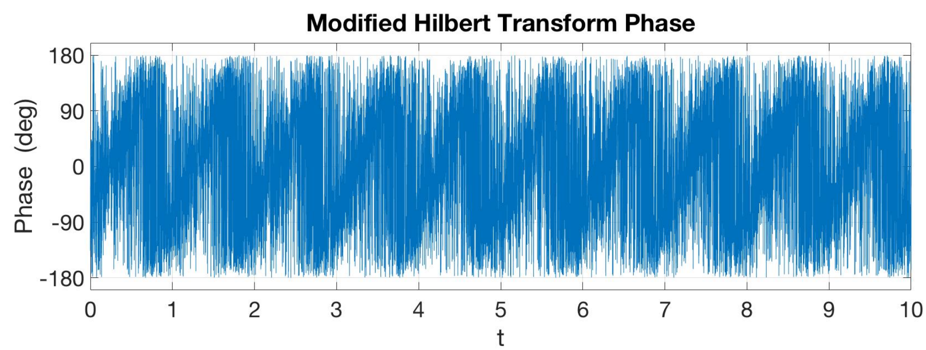

Figure 3.15: Modified Hilbert transform phase from signal of SNR -6dB

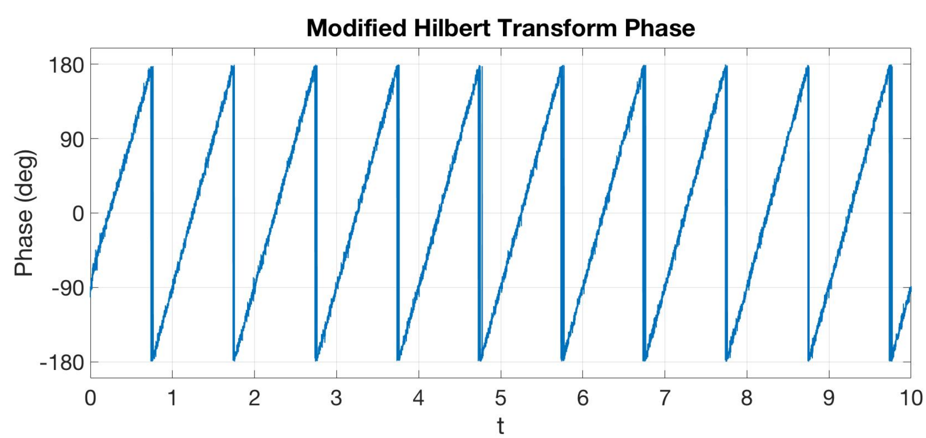

Figure 3.16: Modified Hilbert transform phase of signal with SNR = 20dB

Table 3.2: Various parameters of phase plot obtained when white Gaussian noise was added (Value in degrees) via Modified Hilbert transform

	SNR value (in dB)					
	-6	***-3***	***0***	***+3***	***+10***	***+20***
Starting phase (t=0)	-32.1	-62.1	-51.61	-78.15	-84.61	-88.78
Maximum phase	179.9	179.7	179.8	179.6	179.9	180
Minimum phase	-179.3	-179.4	-179.6	-179.7	-179.9	-180

Other plots behave in same manner as above with the signal becoming more accurate with increasing SNR value towards positive side. All signals were periodic having an approximate time period of 1 sec. The starting phase, the maximum and minimum phase have been given in Table 3.2.

Description

Addition of white Gaussian noise with expected mean equal to zero introduces error in phase calculated from Modified Hilbert transform. Modified Hilbert kernel $\frac{1}{t-k}$ (t is domain in which transform is done and k is the variable with which Hilbert transform is calculated) is multiplied with the input signal which contains noise and is integrated over the whole range of observation. The integration of the noise component does not give value equal to zero and thus, the phase is not well defined, rather jumping instantaneously at every point. The window function and the filter used in Modified Hilbert Transform cannot get rid of the white Gaussian noise. The results still carry small fluctuations even at very high SNR like +20 dB. Thus, Modified Hilbert transform phase in presence of white Gaussian noise is not reliable and vulnerable to fluctuations. The phase moves closer to ideal phase as SNR of signal is increased.

3.4.3 Effect of addition of offset

3.4.3.1 Signal with offset = +3 dB

The phase obtained from Modified Hilbert transform is shown in Fig. 3.17. The phase starts from -87.8 degrees at t=0. The first maximum occurs at t=0.751 sec. The results has a time period of 1 sec. The phase varies from -180 degrees to +179.9 degrees.

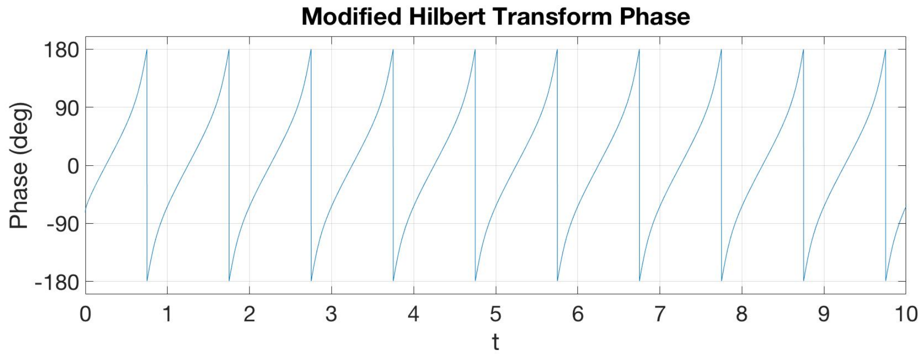

Figure 3.17: Modified Hilbert transform phase with signal having offset = +3dB

Description

This discussion is similar to 3.3.3.4. Fig. 3.17 shows phase obtained when offset is equal to +3 dB. The argument varies from -180 degrees to +180 degrees and the phase is restricted between these two values. The window function and the filter used in Modified Hilbert Transform cannot get rid of the error in phase introduced by offset. When SNR is increased towards more positive side by decreasing the offset, the result starts looking like the ideal phase.

3.4.4 Effect of change in window length

3.4.4.1 Multiplying factor = 4

The phase obtained from Modified Hilbert transform is shown in Fig. 3.18. The phase starts from -90 degrees at t=0. The first maxima comes at t=0.75 sec corresponding to 180 degrees. The resulting waveform is periodic having a time period of 1 sec. The phase varies from -180 degrees to +180 degrees.

3.4.4.2 Multiplying factor = $\frac{1}{4}$

The phase obtained from Modified Hilbert transform is shown in Fig. 3.19. The phase starts from -89.89 degrees at t=0. The resulting waveform is periodic having a time period of 1 sec. There are large deviations from the original phase. The phase varies from -180 degrees to +180 degrees.

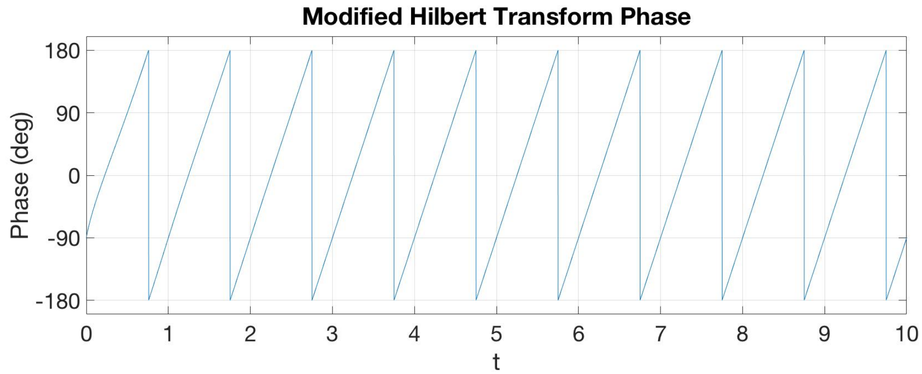

Figure 3.18: Modified Hilbert transform phase with multiplying factor = 4

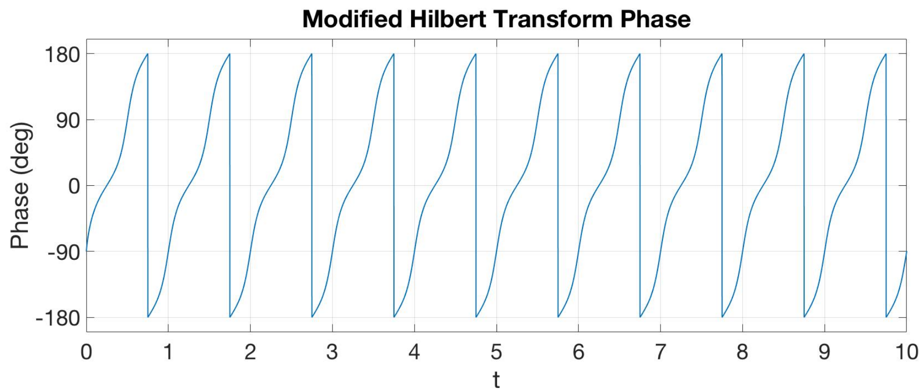

Figure 3.19: Modified Hilbert transform phase with multiplying factor = 0.25

Table 3.3: Various parameters of phase plot obtained when window length was varied (Value in degrees) via Modified Hilbert transform

	Multiplying factor			
	4	***2***	***0.5***	***0.25***
Starting phase (t=0)	-90	-90	-90	-89.88
Maximum phase	180	180	180	180
Minimum phase	-180	-180	-180	-180
Deviation from ideal phase	Least	Small	High	Highest

Description

According to the conducted experiment, window length should ideally be two periods long. The multiplying factor is multiplied with the length of the ideal window. A length of half a period is insufficient to extract out phase from the signal as shown in Fig. 3.18. A window length of two is optimum for the extraction of phase of the signal as shown in Fig. 3.19.

3.4.5 Effect of phase discontinuity

The phase obtained by modified Hilbert transform from the input signal having a single phase discontinuity (input signal is discussed in 3.2.6) is shown in Fig.3.20.

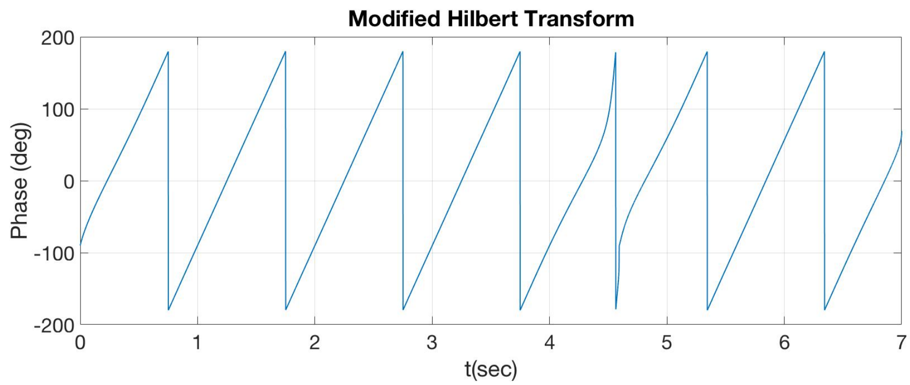

Figure 3.20: Phase estimation from modified Hilbert transform

3.5 Short time Fourier transform

3.5.1 Modulated wavelets

The window function is modulated according to the input signal. The modulated wavelets are shown in Fig. 3.21 and Fig. 3.22

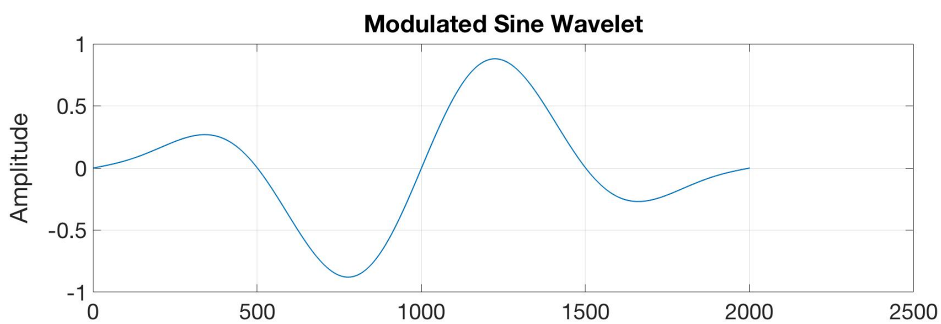

Figure 3.21: Modulated sine wavelet $w_s(m) = w(m) \cdot sin(2\pi(fc/rate) \cdot m)$

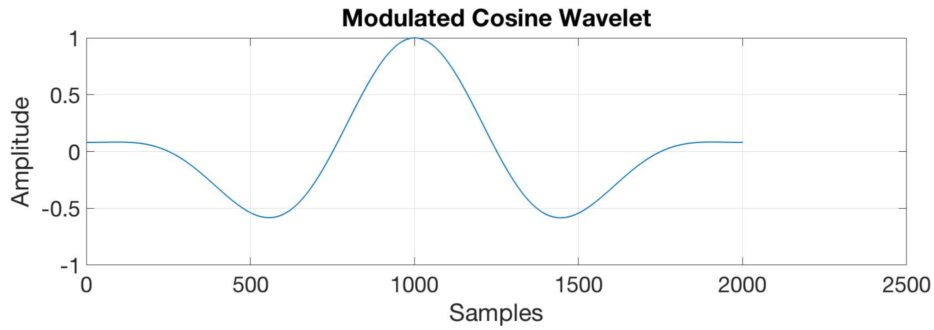

Figure 3.22: Modulated cosine wavelet $w_c(m) = w(m) \cdot cos(2\pi(fc/rate) \cdot m)$

3.5.2 Ideal conditions with no type of noise added

3.5.2.1 Phase estimation

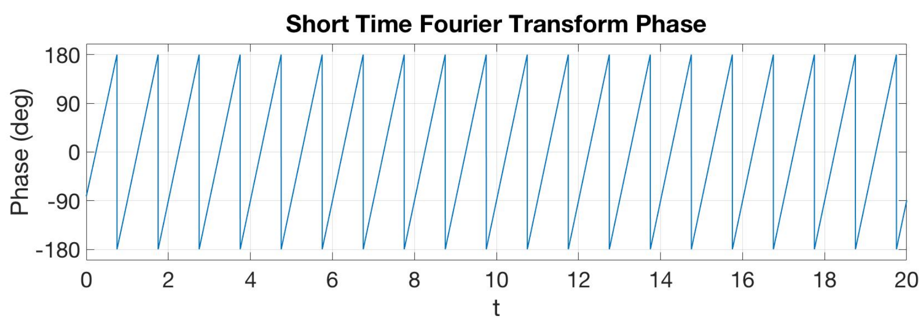

Figure 3.23: Phase estimation via windowed transform method

The phase obtained from short time Fourier transform (STFT) / Windowed transform is shown in Fig. 3.23. The phase is equal to -90 degrees at t=0. The curve attains its first maxima at t=0.75 sec corresponding to π. This is a cyclic curve with a period of 1 sec.

3.5.3 Effect of addition of white Gaussian noise

3.5.3.1 Signal with SNR = -6 dB

The phase obtained from STFT is shown in Fig. 3.24. The phase starts from -89 degrees at t=0. The curve attains its first maxima at t=0.75 sec corresponding to a phase value of π. This is a cyclic curve with a period of 1 sec. The noise has a negligible effect on the phase and looks similar to Fig. 3.20.

3.5.3.2 Signal with SNR = +20 dB

The phase obtained in this case is shown in Fig. 3.25. The phase starts from -89.9 degrees at t=0. The curve attains its first maxima at t=0.75 sec corresponding to π. This is a cyclic curve with a period of 1 sec. The noise has negligible effect on the phase and looks similar to the phase evaluated in the noise free case as shown in Fig. 3.24.

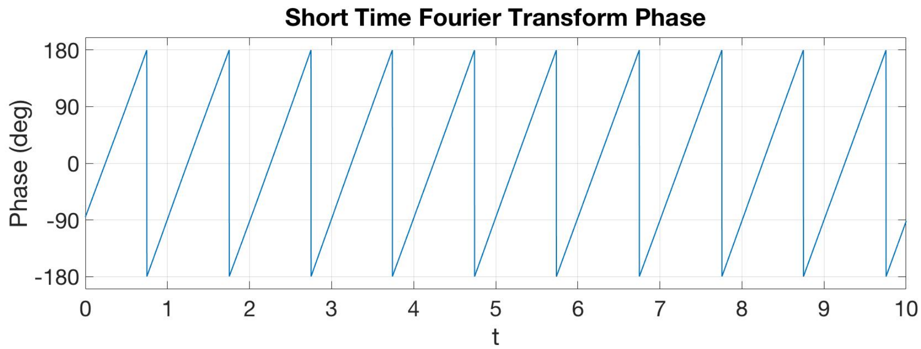

Figure 3.24: STFT phase with a signal a SNR of -6 dB. The noise does not have any impact on phase extraction.

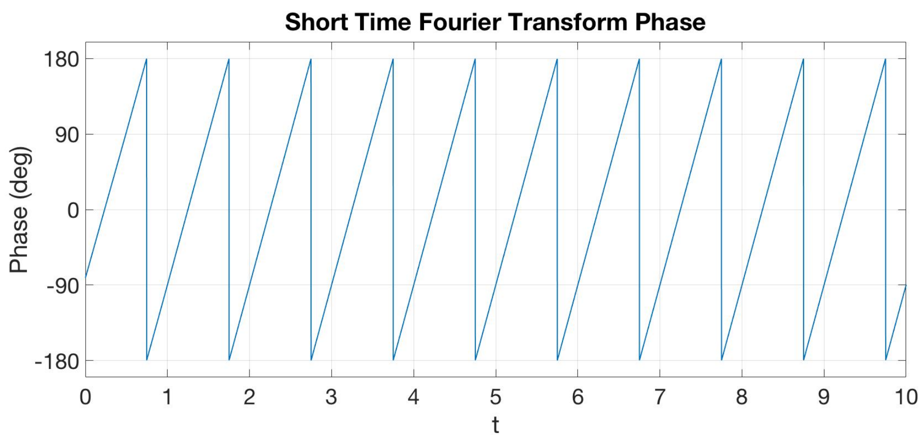

Figure 3.25: STFT phase of signal with a SNR of +20 dB

Table 3.4: Various parameters of phase plot obtained when white Gaussian noise was added (value in degrees) via STFT

	SNR value (in dB)					
	-6	***-3***	***0***	***+3***	***+10***	***+20***
Starting phase (t=0)	-89	-89.7	-89.8	-89.9	-89.9	-89.9
Maximum phase	180	180	180	180	180	180
Minimum phase	-180	-180	-180	-180	-180	-180

Other plots behave in same manner as above with the signal remaining always accurate. All signals were periodic having a time period of 1 sec and look similar to the ideal phase. The starting phase, the maximum and minimum phase have been given in Table 3.4.

Description

The real and imaginary parts of the STFT kernel $e^{-j\omega n}$ (according to Euler's theorem is equal to *cos(ωn) - jsin(ωn)*) is multiplied with the input signal (which contains noise) and is integrated over the whole range of observation. Since sin and cos functions are periodic, and integration of a periodic function multiplied with zero mean noise is equal to zero. Thus, there is negligible effect of addition of white Gaussian noise on STFT phase. If, however, the mean of noise was not equal to zero, the phase would have been different than the ideal phase.

3.5.4 Effect of addition of offset

3.5.4.1 Signal with offset = -10 dB

The phase obtained from STFT is shown in Fig. 3.26. The phase starts from -67.57 degrees at t=0. The curve attains its first maxima at t=0.75 sec corresponding to π. This is a cyclic curve with a period of 1 sec. The offset has no considerable effect on the phase.

3.5.4.2 Signal with offset = +3 dB

The phase obtained from STFT is shown in Fig. 3.27. The phase starts from -89.99 degrees at t=0. The curve attains its first maxima at t=0.75 sec corresponding to π. This is a cyclic curve with a period of 1 sec. The phase of the signal varies from -180 degrees to 180 degrees. The result looks similar to the ideal phase obtained from STFT.

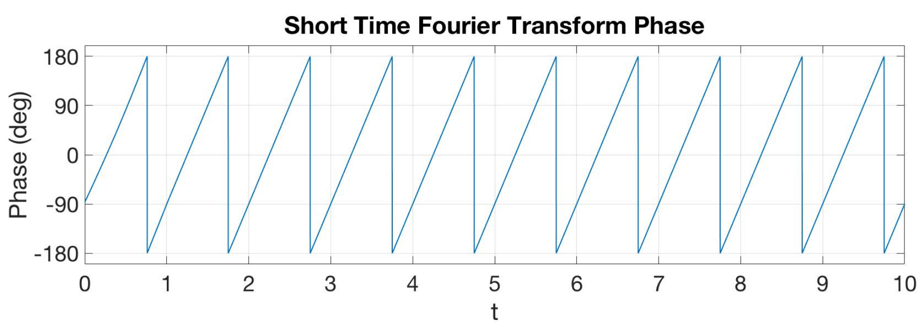

Figure 3.26: STFT phase with signal having an offset = -10 dB

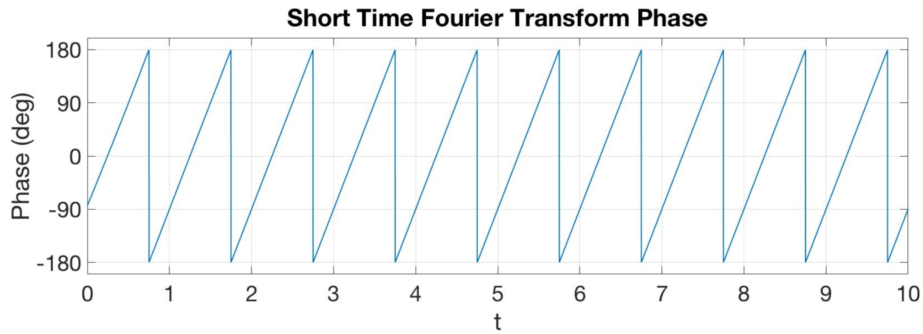

Figure 3.27: STFT phase with signal having offset = +3 dB

Description

The real and imaginary parts of the STFT kernel $e^{-j\omega n}$ (according to Euler's theorem is equal to *cos(ωn) - jsin(ωn)*) is multiplied with the input signal (which contains noise) and is integrated over the whole range of observation. Since sine and cosine are periodic functions and, the integration of a periodic function over a period is equal to zero. Thus, there is negligible effect of the offset on STFT phase estimation.

3.5.5 Effect of change in window length

3.5.5.1 Multiplying factor = 4

The phase obtained from STFT is shown in Fig. 3.28. The phase starts from -90 degrees at t=0. The first maxima comes at t=0.75 sec corresponding to 180 degrees. The resulting waveform is periodic having a time period of 1 sec. The phase varies from -180 degrees to +180 degrees.

3.5.5.2 Multiplying factor = $\frac{1}{4}$

The phase obtained from STFT is shown in Fig. 3.29. The phase starts from -42.02 degrees at t=0. The results are deviant from ideal phase. The first maxima comes at t=0.75 sec corresponding to 180 degrees. The resulting waveform is periodic having a time period of 1 sec. The phase varies from -180 degrees to +180 degrees.

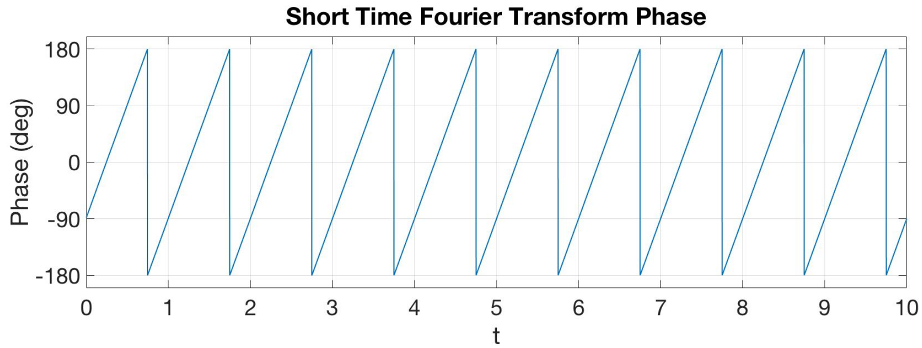

Figure 3.28: STFT phase with multiplying factor = 4

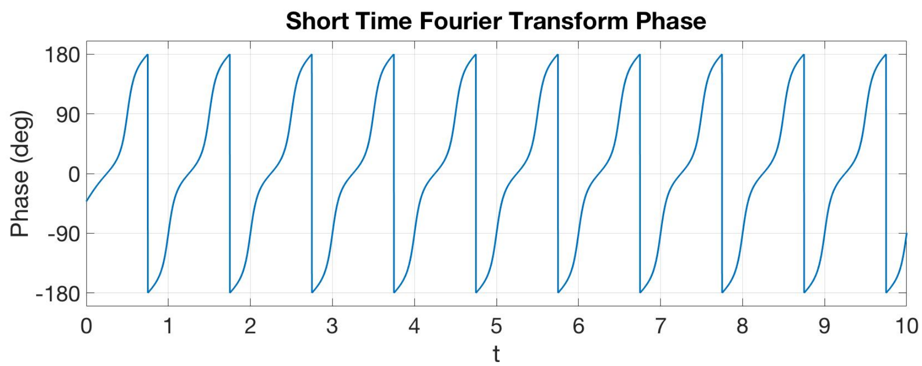

Figure 3.29: STFT phase with multiplying factor = 0.25

Description

This discussion is same as in section3.4.4.2

3.5.6 Effect of wrong estimation of frequency of interest in the input signal

The noise free input (noise free) had 1000 samples in 1 sec as the frequency was 1 Hz and sampling rate was 1000. In this section, we pretend that the central frequency of the input signal was not 1 Hz but rather 2 Hz, 1 Hz and 0.5 Hz. The different values are considered in terms of samples (as sampling rate is kept fixed). Other period values (between 500 samples and 2000 samples with a difference of 100 each) are not shown in this section because they exactly replicate the phase with one plot or the other shown below. The output with 500 samples is shown in Fig. 3.30. The output with 1000 samples is shown in Fig. 3.31. The outputs corresponding to 600 samples upto 1900 samples look similar. Thus, they are not plotted in the report. The output with 2000 samples (twice the signal's frequency) is shown in Fig. 3.32.

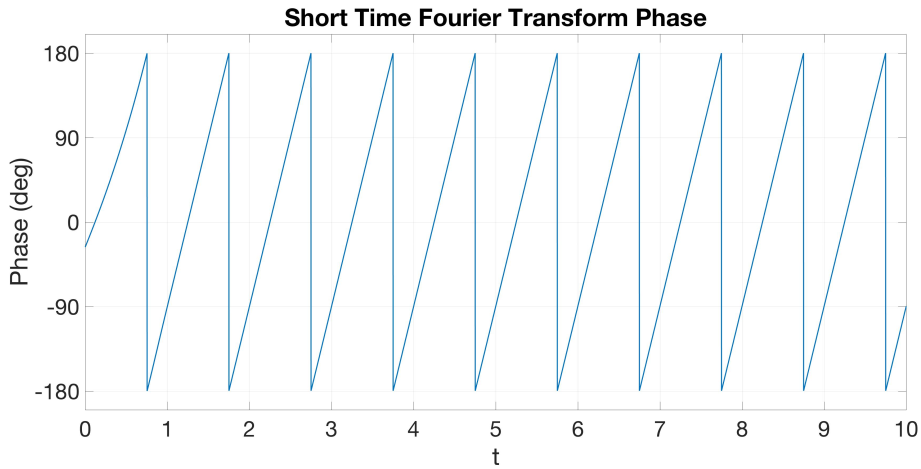

Figure 3.30: STFT phase when samples were taken equal to 500

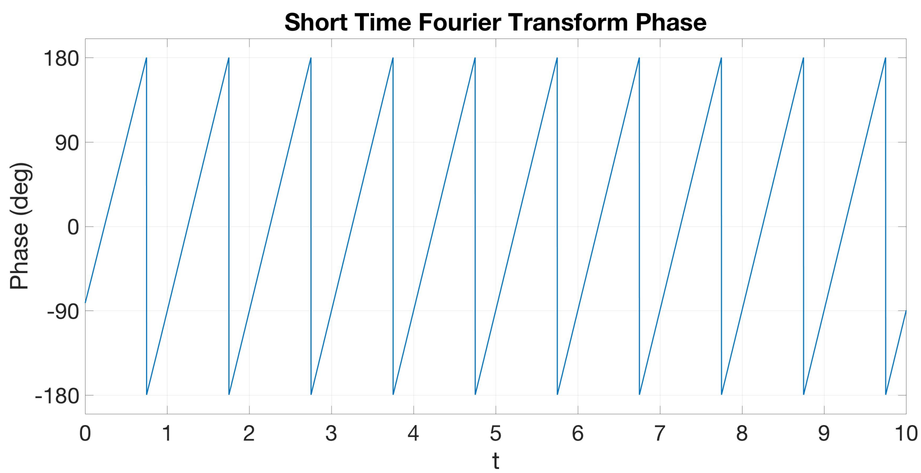

Figure 3.31: STFT phase when samples were taken equal to 1000

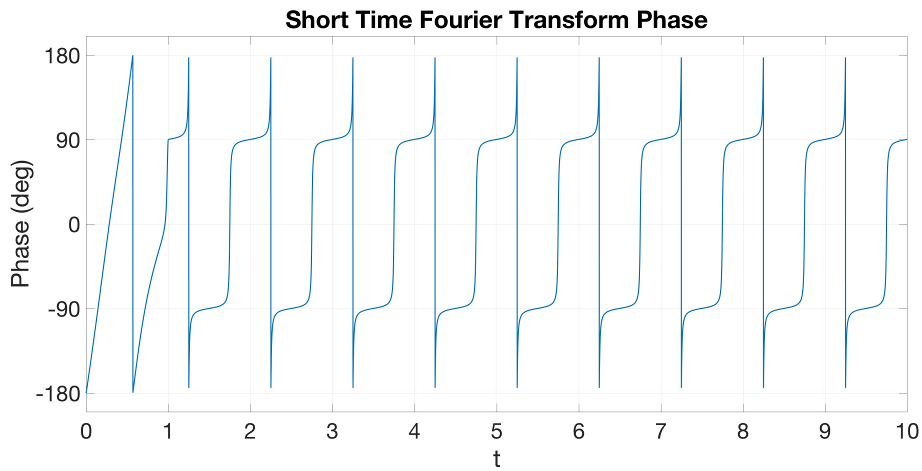

Figure 3.32: STFT phase when samples were taken equal to 2000

Description

The implementation of the method involves convolution of the windowed wavelet with the input signal in time domain. The time domain windowed wavelet(s) are shown in Fig. 3.21 and Fig. 3.22. The input signal is an impulse signal in frequency domain. Therefore, the multiplication of the input signal with windowed wavelet in frequency domain yields the signal whose phase is to be calculated. The change in number of samples correspond to shifting of input impulse signal in frequency domain along frequency axis. Various samples therefore give different phase. Samples from 600 to 1400 looks similar to each other because these frequencies (samples) come inside the passband of the filter. The band pass filter is not able to band pass the frequency components corresponding to 500 samples and 2000 samples completely and, therefore, the phase obtained in these cases is deviant from ideal phase as shown in Fig. 3.30 and Fig. 3.32 respectively. Band pass filter represented by STFT method can look similar to one shown in Fig. 3.33. G(f) represents the transfer function of the band pass filter and the colourful impulse signals in Fig. 3.30 represents the input signal at different frequency points.

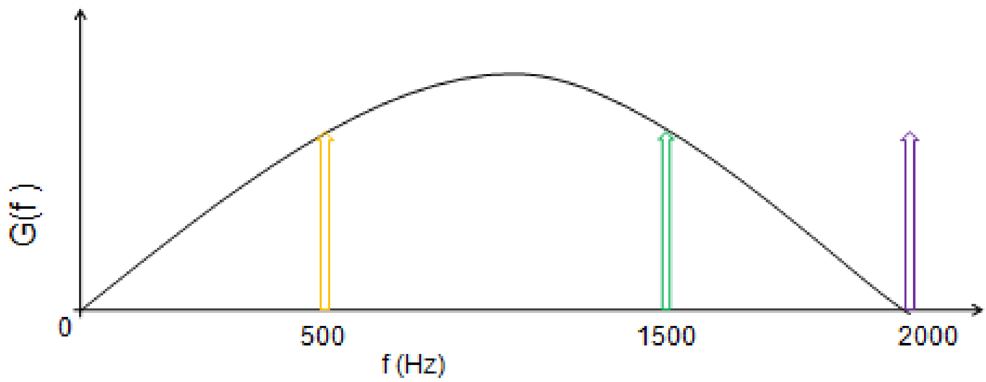

Figure 3.33: Band pass filter represented by STFT in frequency domain along with the impulse input at various instants of frequency. G(f) represents the transfer function of this filter in frequency domain

3.5.7 Effect of phase discontinuity

The phase obtained by STFT from the input signal having a single phase discontinuity (input signal is discussed in 3.2.6) is shown in Fig.3.34.

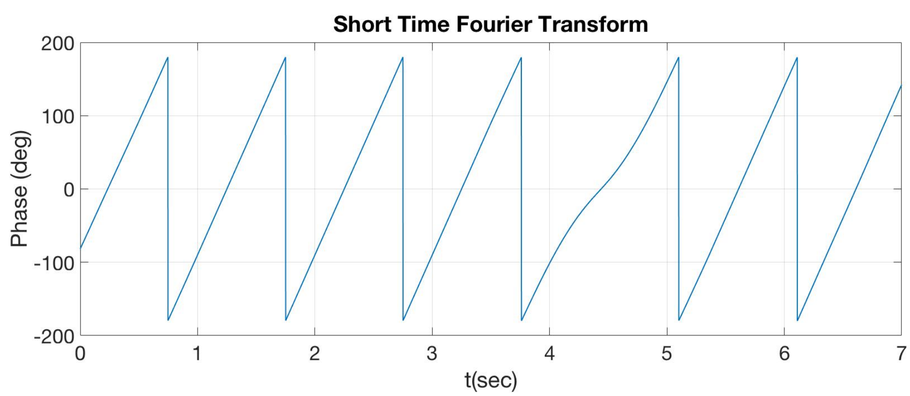

Figure 3.34: Phase estimation from Short time Fourier transform

3.6 Rantner method

3.6.1 Ideal conditions with no type of noise added

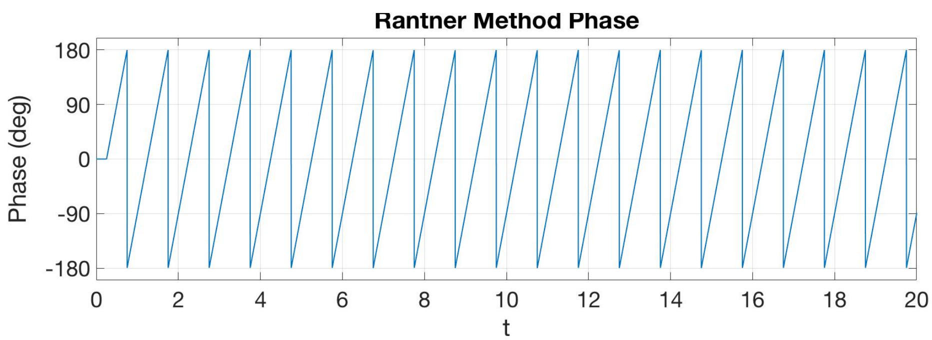

Figure 3.35: Phase obtained from Rantner method (taken from Wilhelm's thesis)

The method gives phase that starts varying from t=0.25 sec (1/4 period of the input signal) because, in this method implementation, the signal was delayed by a value equal to 1/4 the period of the signal i.e. 0.25 sec. The first 0.25 sec do not convey the phase carried by the input signal. The first maximum comes at t=0.75 sec corresponding to a phase of π. The next peak comes at t=1.75 sec and the cycle repeats itself with a time period of 1 sec.

3.6.2 Effect of addition of white Gaussian noise

3.6.2.1 Signal with SNR = -6 dB

The phase obtained from Rantner method is shown in Fig. 3.36. The phase starts from -180 degrees at t=0.249 sec and it instantaneously falls to 2.3 degrees at t=2.51 sec. The results are sporadic. The phase varies from -179.7 degrees to +180 degrees and is periodic with time period of 1 sec. There are many phase jumps at every point and thus, the phase information is not reliable.

3.6.2.2 Signal with SNR = 20 dB

The phase obtained from Rantner method is shown in Fig. 3.37. It starts giving results from t=0.25 sec (1/4 period of the input signal). The waveform looks similar to ideal phase with a time period of 1 sec. The phase varies from -180 degrees to +180 degrees. Despite such SNR value, the disturbance in phase can be observed clearly. Thus, Rantner method is not the best method for phase evaluation when artefact is white Gaussian noise.

Table 3.5: Various parameters of phase plot obtained when white Gaussian noise was added (Value in degrees) via Rantner method

	SNR value (in dB)					
	-6	*-3*	*0*	*+3*	*+10*	*+20*
Starting phase (t=0.25 sec)	2.3	15.4	18.5	23.5	14.9	9.5
Maximum phase	180	179.4	180	179.6	180	180
Minimum phase	-179.7	-179.9	-179.9	-179.7	-179.9	-180

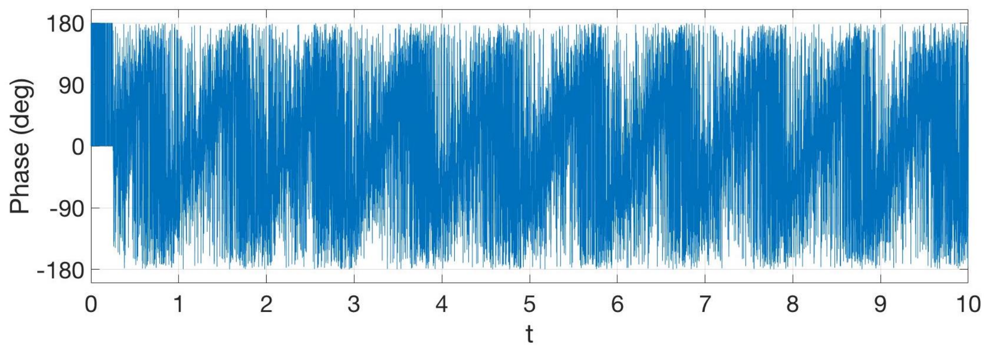

Figure 3.36: Rantner method phase of signal of SNR -6 dB

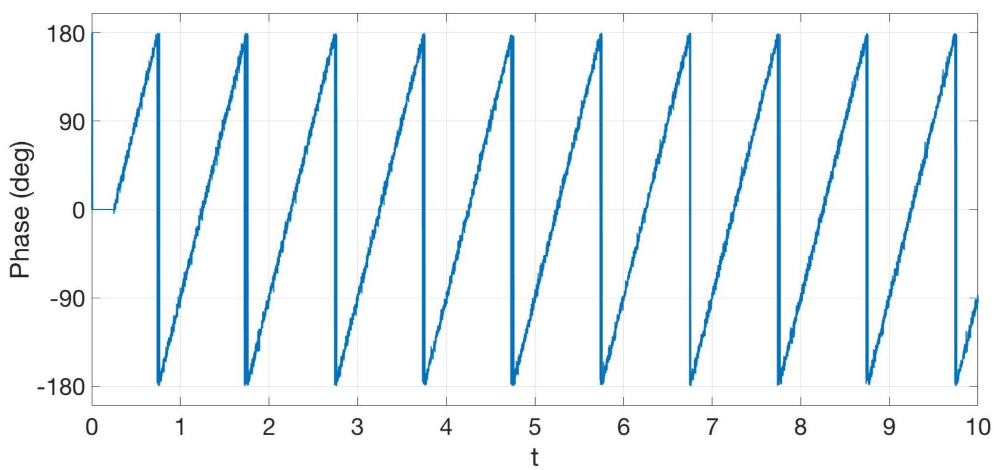

Figure 3.37: Rantner method phase of signal of 20 dB

Other plots behave in same manner as above. All signals were periodic having an approximate time period of 1 sec. The starting phase, the maximum and minimum phase have been given in Table 3.5.

Description

Addition of white Gaussian noise with an expected value equal to zero introduces error in phase calculated from Rantner method. This method involves a delay of the signal and then, calculating the phase difference with the help of delayed signal. Like the input signal, the delayed signal is also a noisy signal. Therefore, the phase is not independent of noise. Results become less noisy with increasing SNR value towards positive SNR values.

3.6.3 Effect of addition of offset

3.6.3.1 Signal with offset = -10 dB

The phase obtained from signal with offset = -10 dB from Rantner method is shown in Fig. 3.38. The phase starts varying from t=0.25 sec (1/4 period of the input signal). The phase of the signal varies from 32.90 degrees to 62.45 degrees. Since the deviation from the ideal phase is very large and the span of phase variation is just approximately 30 degrees (instead of 360 degrees in the ideal case), the results are highly erratic and are not reliable.

3.6.3.2 Signal with offset = +3 dB

The phase obtained from signal with offset = +3 dB from Rantner method is shown in Fig. 3.39. The phase starts giving results from t=0.25 sec (1/4 period of the input signal). The result is periodic with a time period of 1 sec, and the first peak occurs at 0.75 sec. The phase varies from -180 to 180 degrees.

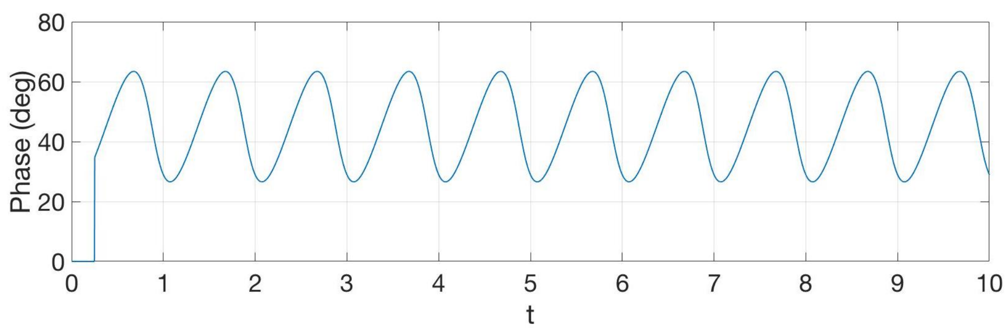

Figure 3.38: Rantner method phase with signal having offset = -10 dB

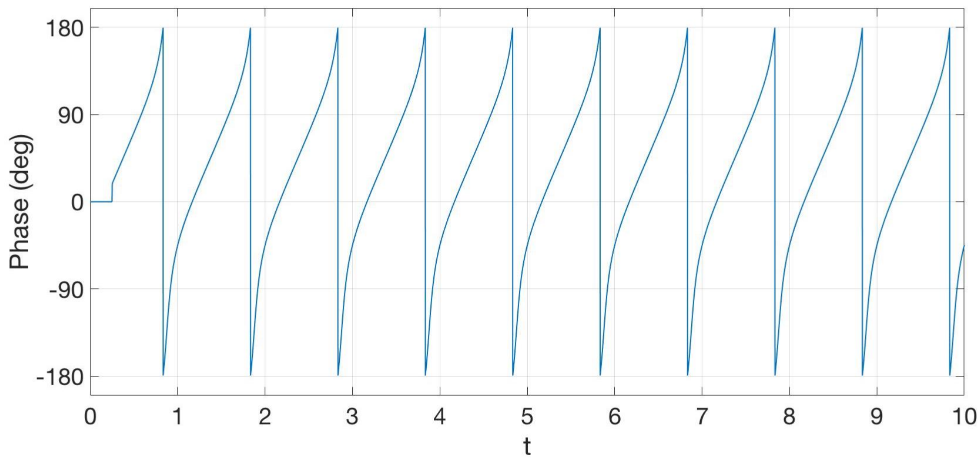

Figure 3.39: Rantner method phase with signal having offset = +3 dB

Description

For first 0.25 sec (time with which signal is delayed), the phase is constant and does not respond to change in amplitude in input signal, as expected. The plot between the delayed signal and the input signal is a circle centered at origin with radius equal to the amplitude of the input signal. As discussed in section 3.3.3.4, addition of an offset shifts the center of the circle towards positive X axis. The phase, at a particular time, of a point is the principal argument of that point in the XY plane. Larger the value of offset, the span (range) of the phase decreases proportionally as shown in Fig. 3.34. Decreasing the magnitude of offset, increases the range of phase. The phase starts looking like ideal phase, as we decrease the value of the offset.

3.6.4 Effect of phase shift on Rantner method

3.6.4.1 Delay of $\pi/4$

The phase obtained by delaying the signal from $\pi/4$ (1/8 period of the input signal) is shown in Fig. 3.40. The first maxima comes at t=0.75 sec and phase corresponding to π. The next peak comes at t=1.75 sec and the cycle repeats with a time period of 1 sec. The phase obtained does not follow ideal phase. This phase shift when operated inside inverse tangent function, produces wrong results in the measurement of the phase of the signal.

3.6.4.2 Delay of π

The phase obtained by delaying the signal from π is shown in Fig. 3.41. The results are quite different from the ideal phase and thus, this method completely fails in these conditions. The cycle repeats with time period of 1 sec. The phase predominantly varies from -45 degrees to

+135 degrees. Change of π corresponds to half a period of the signal. A half period delay of the sine signal corresponds to the inverted sine signal. The function inside inverse tangent function, according to Rantner method, is therefore -1. Thus, the phase plot only gives -45 degrees or +135 degrees as a result.

3.6.4.3 Delay of 2π

The phase obtained by delaying the signal from 2π is shown in Fig. 3.42. The phase predominantly varies from -135 degrees to +180 degrees. 2π radian change corresponds to one period of the signal. One period delay of the sine signal is again a shifted sine signal. The function inside inverse tangent function, according to Rantner method, is therefore 1. Thus phase plot only give +45 or -135 degrees as a result. The starting time values (0 to 2 sec) which do not have the second signal has zero phase and does not convey any phase information. The inverse tangent of zero is 0 degrees or +180 degrees and, therefore, for the starting 2 sec, the phase jumps between 0 and +180 degrees. The reason behind random jumps at some random points in Fig. 3.37 and Fig. 3.38 is unknown and needs more research into the topic.

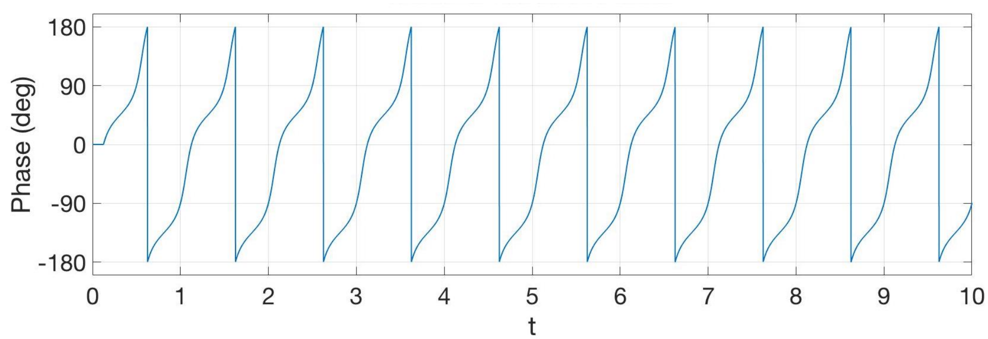

Figure 3.40: Effective delay of $\pi/4$

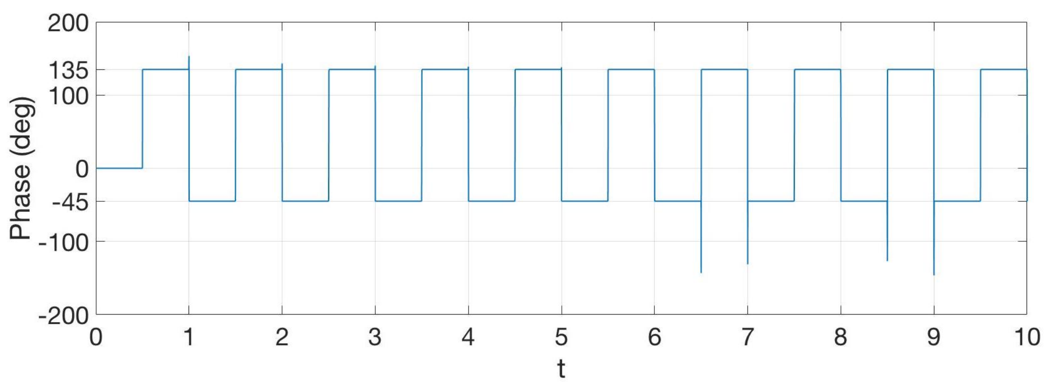

Figure 3.41: Effective delay of π

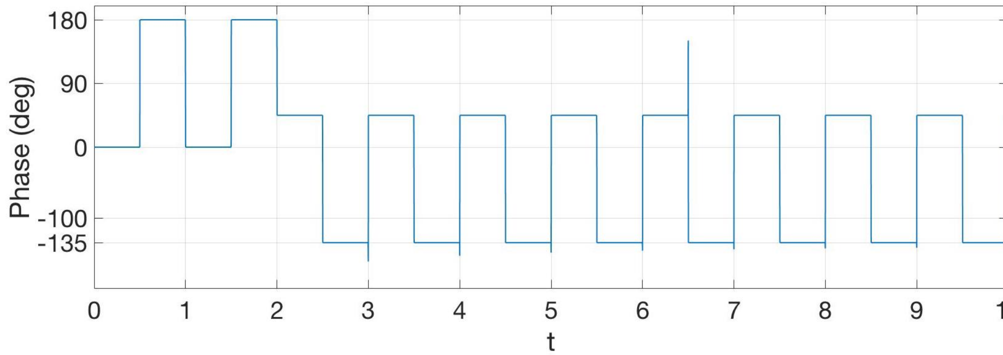

Figure 3.42: Effective delay of 2π

3.6.5 Effect of phase discontinuity

The phase obtained by Rantner method from the input signal having a single phase discontinuity (input signal is discussed in 3.2.6) is shown in Fig.3.52.

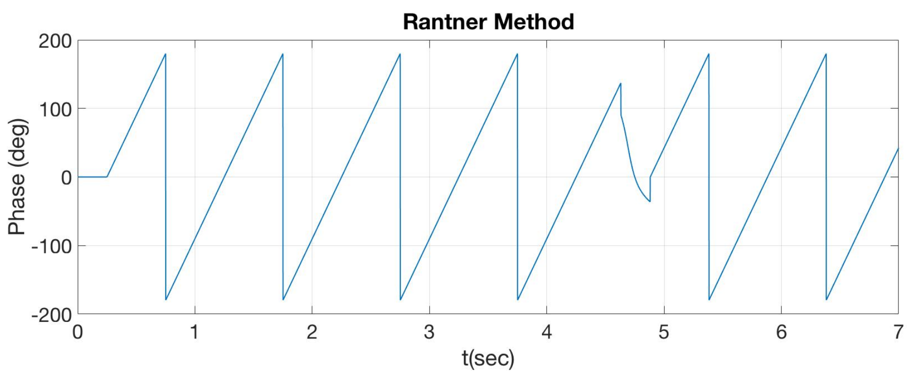

Figure 3.43: Phase estimation from Rantner method

3.7 Sinusoidal recomposition

3.7.1 Ideal conditions with no type of noise added

The phase obtained from Sinusoidal recomposition is shown in Fig. 3.44. The phase starts from 12 degrees at t=0. The first maximum peak comes at t=0.5 sec corresponding to 179.8 degrees. The next peak comes at t=1.5 sec corresponding to 179.8 degrees. The cycle repeats itself and is therefore periodic with a time period of 1 sec.
The phase varies from -180 degrees to +179.8 degrees.

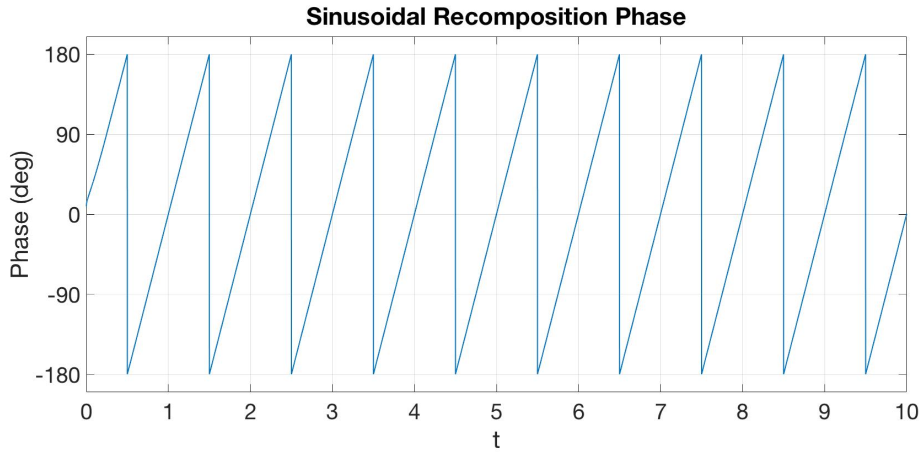

Figure 3.44: Phase estimation from Sinusoidal recomposition method

3.7.2 Effect of addition of white Gaussian noise

3.7.2.1 Signal with SNR = -6 dB

The phase obtained from Sinusoidal recomposition is shown in Fig. 3.45. The phase starts from 24.1 degrees at t=0. The results are sporadic and have a very large deviation from the ideal phase and, thus are not at all reliable and reproducible.

3.7.2.2 Signal with SNR = +20 dB

The phase obtained from Sinusoidal recomposition is shown in Fig. 3.46. The phase starts from 24.1 degrees at t=0. This result looks like the ideal phase.

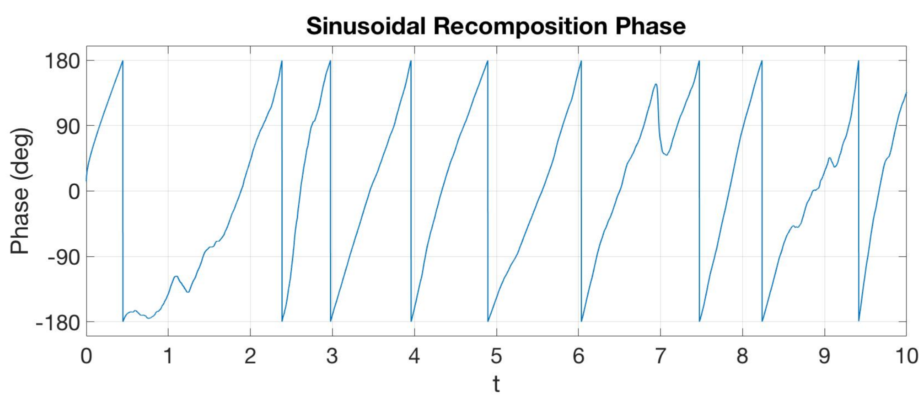

Figure 3.45: Sinusoidal recomposition phase from signal of SNR -6dB

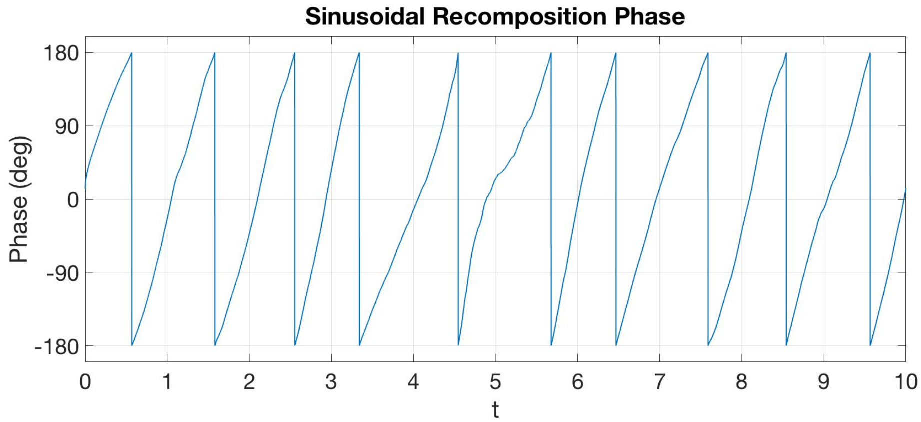

Figure 3.46: Sinusoidal recomposition phase from signal of SNR 20 dB

Table 3.6: Various parameters of the phase plot obtained when white Gaussian noise was added (value in degrees) via Sinusoidal recomposition

	SNR value (in dB)					
	-6	***-3***	***0***	***+3***	***+10***	***+20***
Starting phase (t=0)	24.1	14.1	38.9	18.54	14.3	24.1
Maximum phase	179.7	180	180	180	180	180
Minimum phase	-179.4	-180	-180	-180	-180	-180

Other plots behave in same manner as above with the signal becoming more accurate with increasing SNR value towards positive SNR values. The starting phase, the maximum and minimum phase have been given in Table 3.6.

Description

The filter used in sinusoidal recomposition is a band pass filter (in frequency domain) which only allow certain frequencies to pass and attenuate all other information content. White Gaussian noise has a constant spectral amplitude, and, the frequencies lying out of the pass band get attenuated but not become equal to zero. Therefore, the error due to these frequencies is still retained in the output phase signal. However, there are no distortions in the phase because that is filtered out effectively by the filter.

3.7.3 Effect of addition of offset

3.7.3.1 Signal with offset = -3 dB

The phase obtained from sinusoidal recomposition is shown in Fig. 3.47. The phase starts from 0.3 degrees at t=0. The curve attains its first maxima at t=0.5 sec corresponding to π. This is a cyclic curve with a period of 1 sec. The result is an exact reproduction of the ideal phase. The offset has no considerable effect on the phase. The phase of the signal varies from -180 degrees to 180 degrees.

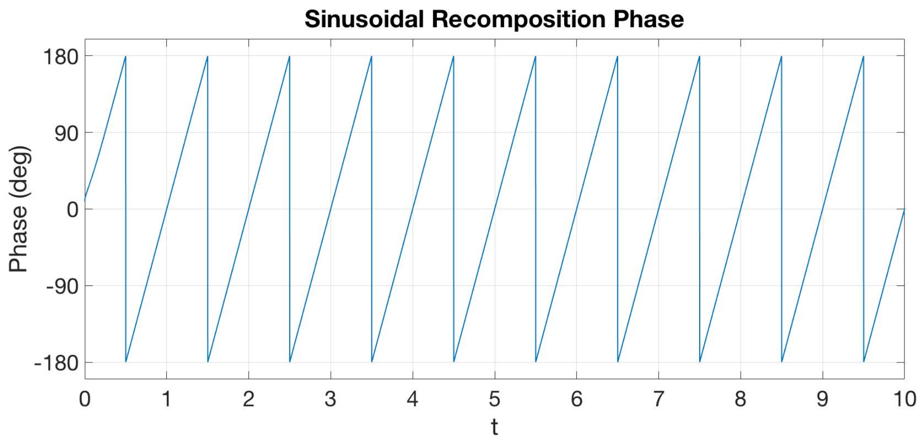

Figure 3.47: Sinusoidal recomposition phase with signal having offset = -3dB

Description

There is no effect of an offset on the phase calculated with sinusoidal recomposition. All other results are not plotted because they exactly look similar to the above figure and gives the ideal phase. The first step in sinusoidal recomposition is the differentiation of the signal. Since offset is a constant artefact, the differentiation of a constant term is zero. Thus, there is no effect of an offset on the determination of the phase of the signal with sinusoidal recomposition.

3.7.4 Effect of wrong estimation of frequency of interest in the input signal

The ideal input (noise free) had 1000 samples in 1 sec because input frequency was 1 Hz and sampling rate was 1 kHz. Sinusoidal recomposition took the number of samples in 1 sec as an input parameter. In this section, we pretend that the central frequency of the input signal was not 1 Hz but rather 2 Hz, 1 Hz and 0.5 Hz. The different values are considered in terms of samples (as sampling rate is kept fixed). Other period values (between 500 samples and 2000 samples with a difference of 100 each) are not shown in this section because they exactly replicate the phase with one plot or the other shown below. The output corresponding to 500 samples is shown in Fig. 3.48. The output corresponding to 1000 samples is shown in Fig. 3.49. The outputs corresponding to 600 samples upto 1900 samples look similar to Fig. 3.49. Thus, they are not plotted in the report. The output corresponding to 2000 samples is shown in Fig. 3.50.

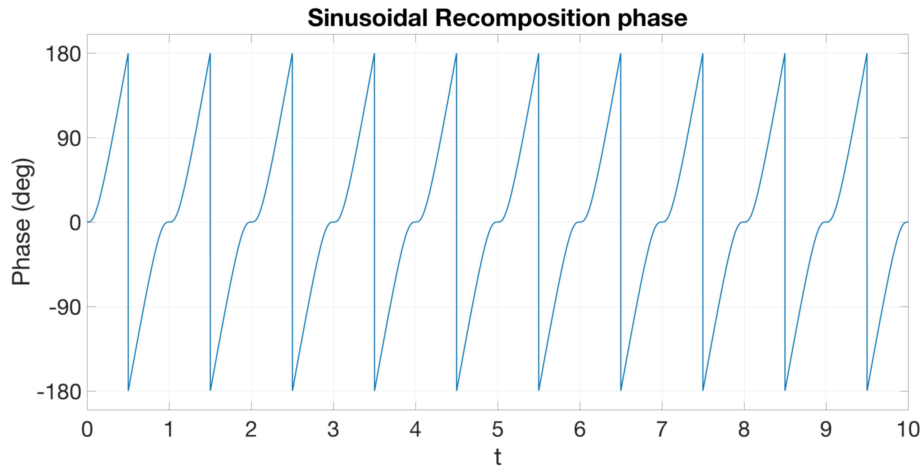

Figure 3.48: Sinuosoidal recomposition phase when one period was assumed to be 500 samples long

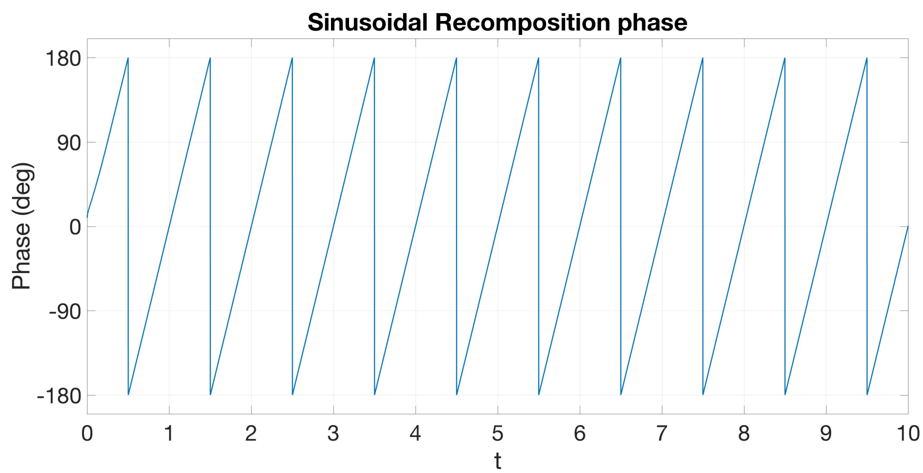

Figure 3.49: Sinuosoidal recomposition phase when one period was assumed to be 1000 samples long

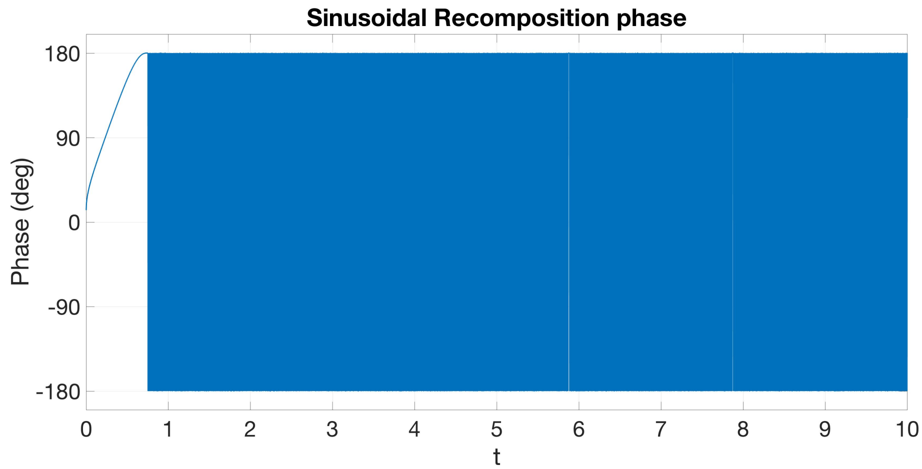

Figure 3.50: Sinuosoidal recomposition phase when one period was assumed to be 2000 samples long

Description

The filter utilized in sinusoidal recomposition method is a band pass filter in frequency domain. The band of frequencies varies from 0 Hz to twice the frequency of the signal. The input signal was an impulse signal (mono frequency component) in frequency domain. Therefore, phase looks like the ideal phase when samples were between 500 to 1900. However, each result has a different offset (phase at t=0) w.r.t one another. When samples were assumed to have a period having 2000 samples, filter did not allow the signal to pass the signal's information content and therefore, the result shown in Fig. 3.50 does not convey about any phase information. Band pass filter represented by sinusoidal recomposition method can look similar to one shown in Fig. 3.33. G(f) represents the transfer function of the band pass filter and the colourful impulse signals in Fig. 3.46 represents the input signal at different frequency points.

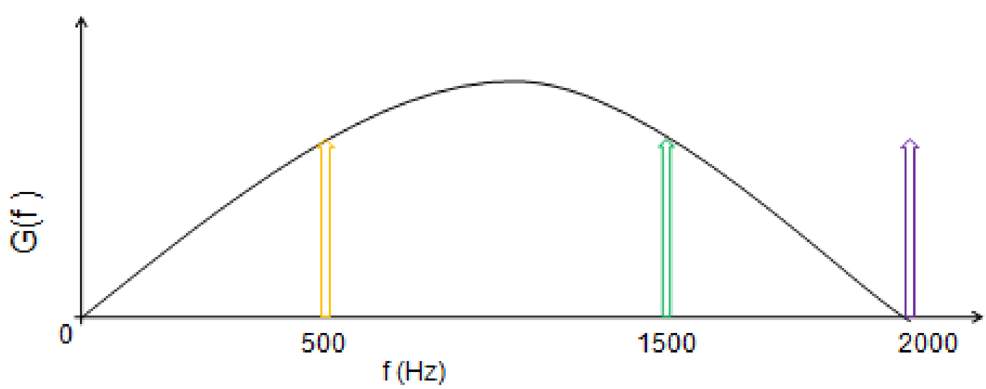

Figure 3.51: Band pass filter represented by sinusoidal recomposition in frequency domain along with the impulse input at various instants of frequency. G(f) represents the transfer function of this filter in frequency domain

3.7.5 Effect of phase discontinuity

The phase obtained by sinusoidal recomposition from the input signal having a single phase discontinuity (input signal is discussed in 3.2.6) is shown in Fig.**??**.

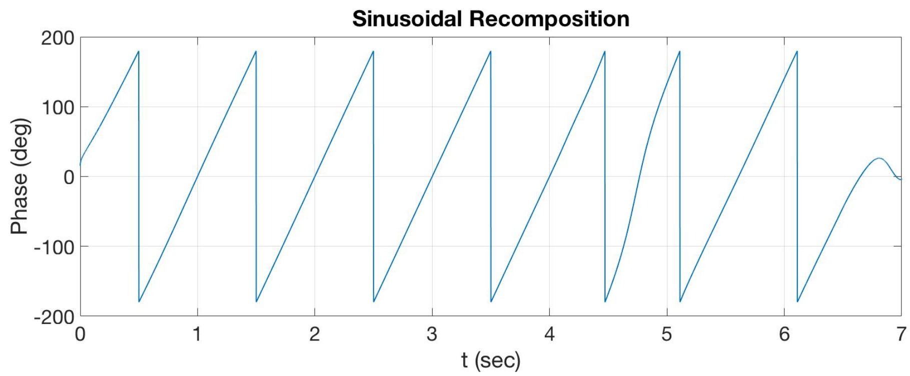

Figure 3.52: Phase estimation from sinusoidal recomposition

CHAPTER 4

Conclusion

Different type of noises were added to the sinusoidal signal and the results were analysed in accordance with three performance indices namely, correlation coefficient, mean of error and standard deviation of error. This section provides the best estimation method when noise is added to the signal according to results illustrated in tables 4.1 - 4.16.

Table 4.1: Correlation coefficients obtained from various methods with addition of white Gaussian noise

	SNR (Value in dB)						
Name of the method	***-6 dB***	***-3 dB***	***0 dB***	***+3 dB***	***+10 dB***	***+20 dB***	***Average***
Hilbert transform	0.276	0.370	0.501	0.631	0.835	0.947	0.593
Modified Hilbert transform	0.279	0.381	0.504	0.627	0.831	0.945	0.595
Short time Fourier transform	0.958	0.975	0.980	0.985	0.992	0.997	**0.981**
Rantner method	0.271	0.378	0.498	0.629	0.838	0.946	0.593
Sinusoidal recomposition	0.009	0.077	0.046	0.105	0.201	0.607	0.162

Table 4.2: Mean error (in degrees) obtained from various methods with addition of white Gaussian noise

	SNR (Value in dB)						
Name of the method	***-6 dB***	***-3 dB***	***0 dB***	***+3 dB***	***+10 dB***	***+20 dB***	***Average***
Hilbert transform	0.091	-0.617	0.177	-0.094	-0.105	-0.256	-0.134
Modified Hilbert transform	-0.039	0.053	-0.187	-0.056	-0.143	-0.111	-0.080
Short time Fourier transform	-0.194	-0.170	-0.181	-0.187	-0.183	-0.175	-0.182
Rantner method	0.191	-0.498	0.197	0.213	-0.063	0.020	**0.010**
Sinusoidal recomposition	2.584	-0.895	-5.033	-1.326	1.829	2.504	-0.056

Table 4.3: Standard deviation of error (in degrees) obtained from various methods with addition of white Gaussian noise

	SNR (Value in dB)						
Name of the method	***-6 dB***	***-3 dB***	***0 dB***	***+3 dB***	***+10 dB***	***+20 dB***	***Average***
Hilbert transform	125.058	116.612	103.824	89.199	59.707	33.718	88.019
Modified Hilbert transform	124.587	115.620	103.359	89.712	60.400	34.167	87.974
Short time Fourier transform	29.825	23.131	20.375	17.782	12.355	7.633	**18.571**
Rantner method	125.382	115.855	103.996	89.473	59.062	34.148	87.986
Sinusoidal recomposition	145.911	141.357	144.101	140.366	132.374	92.360	132.745

Table 4.4: Correlation coefficients obtained from various methods with addition of offset

	Offset (Value in dB)					
Name of the method	***-10 dB***	***-6 dB***	***-3 dB***	***0 dB***	***+3 dB***	***Average***
Hilbert transform	0.841	0.877	0.921	1.00	0.987	0.925
Modified Hilbert transform	0.827	0.870	0.916	0.996	0.984	0.919
Short time Fourier transform	0.987	0.986	0.986	0.991	0.992	0.988
Rantner method	0.529	0.527	0.509	0.463	0.342	0.474
Sinusoidal recomposition	1.00	1.00	1.00	1.00	1.00	**1.00**

Table 4.5: Mean error (in degrees) obtained from various methods with addition of offset

	Offset (Value in dB)					
Name of the method	***-10 dB***	***-6 dB***	***-3 dB***	***0 dB***	***+3 dB***	***Average***
Hilbert transform	-0.179	-0.179	-0.179	-0.089	0.0004	-0.125
Modified Hilbert transform	-0.137	-0.099	-0.055	0.076	0.111	0.096
Short time Fourier transform	-0.117	-0.101	-0.014	-0.292	-0.328	0.170
Rantner method	44.714	44.730	44.747	44.770	44.827	44.758
Sinusoidal recomposition	1.81e-14	1.76e-15	-7.6e-15	7.17e-16	-1.49e-16	**1.23e-14**

Table 4.6: Standard deviation of error (in degrees) obtained from various methods with addition of offset

	Offset (Value in dB)					
Name of the method	***-10 dB***	***-6 dB***	***-3 dB***	***0 dB***	***+3 dB***	***Average***
Hilbert transform	93.273	86.119	76.392	51.961	30.914	67.732
Modified Hilbert transform	93.296	86.152	76.458	52.331	31.691	67.756
Short time Fourier transform	16.548	17.270	17.141	13.760	12.780	15.500
Rantner method	99.250	99.949	94.923	93.652	0.342	77.623
Sinusoidal recomposition	4.56e-14	4.32e-14	3.62e-14	3.41e-14	3.18e-14	**3.8e-14**

Table 4.7: Correlation coefficients obtained from various methods with changing window length

	Window length (factor of signal period)				
Name of the method	***0.25***	***0.5***	***2***	***4***	***Average***
Modified Hilbert transform	0.998	0.999	0.992	0.986	0.994
Short time Fourier transform	0.998	0.999	0.999	0.982	**0.995**

Table 4.8: Mean error (in degrees) obtained from various methods with changing window length

	Window length (factor of signal period)				
Name of the method	***0.25***	***0.5***	***2***	***4***	***Average***
Modified Hilbert transform	0.076	0.044	-0.392	-0.326	**-0.149**
Short time Fourier transform	-0.505	-0.028	0.003	-0.402	-0.233

Table 4.9: Standard deviation of error (in degrees) obtained from various methods with changing window length

	Window length (factor of signal period)				
Name of the method	***0.25***	***0.5***	***2***	***4***	***Average***
Modified Hilbert transform	5.194	3.931	12.672	18.364	10.040
Short time Fourier transform	5.325	3.938	2.580	21.473	**8.329**

Table 4.10: Correlation coefficient obtained from different methods with considering wrong estimation of frequency

	No. of Samples				
Name of the method	***500***	***1000***	***1500***	***2000***	***Average***
Short time Fourier transform	0.993	1.00	0.989	-0.776	**0.551**
Sinusoidal recomposition	0.985	1.00	0.999	-0.033	0.735

Table 4.11: Mean error (in degrees) obtained from various methods with considering wrong estimation of frequency

	No. of Samples				
Name of the method	***500***	***1000***	***1500***	***2000***	***Average***
Short time Fourier transform	0.346	0.000	-0.515	-0.650	-0.205
Sinusoidal recomposition	3.45e-04	0.0000	-4.60e-04	0.3229	**0.080**

Table 4.12: Standard deviation of error (in degrees) obtained from different methods with considering wrong estimation of frequency

	No. of Samples				
Name of the method	***500***	***1000***	***1500***	***2000***	***Average***
Short time Fourier transform	11.769	0.000	15.447	184.120	**52.834**
Sinusoidal recomposition	20.988	0.000	1.043	210.234	58.066

Table 4.13: Correlation coefficient obtained from Rantner method with delay introduced

	Delay (in terms of period of signal)			
Name of the method	***1/8***	***1/2***	***1***	***Average***
Rantner method	0.399	-6.02e-04	6.67e-04	**0.132**

Table 4.14: Mean error (in degrees) obtained from Rantner method with delay introduced

	Delay (in terms of period of signal)			
Name of the method	***1/8***	***1/2***	***1***	***Average***
Rantner method	-22.556	44.970	-43.787	**-7.124**

Table 4.15: Standard deviation of error (in degrees) obtained from Rantner method with delay introduced

	Delay (in terms of period of signal)			
Name of the method	***1/8***	***1/2***	***1***	***Average***
Rantner method	112.169	137.406	138.088	**129.221**

Table 4.16: Phase discontinuity performance parameters by various methods

	Parameter recorded		
Name of the method	***Correlation coefficient***	***Mean error (degrees)***	***Standard deviation of error (degrees)***
Hilbert transform	0.938	2.707	**35.352**
Modified Hilbert transform	0.985	-2.049	99.442
Short time Fourier transform	0.972	**-1.802**	99.766
Rantner method	**1.00**	-4.118	98.953
Sinusoidal recomposition	0.993	-3.159	103.548

From the tables, it was observed that the Hilbert transform was the best method when standard deviation was studied in the signal with phase discontinuity. It had least standard deviation. Modified Hilbert transform won over other methods when mean error was taken into study for a variation in window length. STFT won 7 tables out of 15. Method Rantner won in two tables. It had the least mean error in presence of white Gaussian noise and it had the highest correlation coefficient for the estimation in the presence of phase discontinuity in the signal. Sinusoidal recomposition appeared as a clear winner in all three tables for which the offset was added as an artifact. It also performed the best when the mean error was taken into consideration in case of incorrect estimation of frequency of interest in the input signal.

The effect of each method on various noise was different and one method outperformed the other in each case. When white Gaussian noise was added to the signal, STFT worked the best as it won 2 out of 3 tables. When offset was added to the signal, sinusoidal recomposition came out to be the clear winner as it won all the three tables. When window length was varied to extract the phase, STFT again won 2 out of 3 tables. When the frequency of interest in the input signal was interpreted incorrectly, STFT again gave better results than other methods. In case of the presence of a phase discontinuity in the signal, there was no clear winner since each table had a different winner.
Overall, *STFT* appeared as the best transformation when the above listed noise were taken into consideration for an ideal sine wave input signal. It won 7 tables out of 15 and therefore, it stands as the best method for the phase extraction of the signal.
Different noise parameters were added to the input signal and instantaneous phase was extracted out with five methods taken from literature review. The phase obtained from each method in each case (of different noise) was evaluated on the basis of three performance parameters, namely, correlation coefficient, mean of error and standard deviation of error respectively. The best method was identified for each case and an overall winner was declared which was the most reliable method considering all the cases. The best method is not certainly the best solution for each and every case but, is a winner by majority in the results.

CHAPTER 5

Future Scope

This project work is not limited to the results obtained in this report. The horizon of this work can be expanded in a multitude of directions. This report includes perfect sine wave as an input. The effect on phase by a sinusoidal wave with harmonics could be studied. This signal can be composed, for example, of two or more sinusoidals having different frequencies. Afterwards, cardiac and other bio signals can be taken as input signal. Different signals like ECG, intracardiac signals, electrogram signals can be taken into consideration and instantaneous phase can be extracted from them. In addition, a new method can be developed and implemented. This method could be a modified or extended version of an already existing one such as the methods discussed in this report. The new method should be able to give better performing results and could be optimized for different noise parameters.

References

[1] S. Yusuf, S. Reddy, S. Qunpuu, et al., "Global burden of cardiovascular diseases-part i: General considerations, the epidemiologic transition, risk factors, and impact of urbanization," *QUARTERLY JOURNAL OF CARDIOLOGY*, vol. 7, no. 2, pp. 132–139, 2002.

[2] T. Zielinski, "Instantaneous phase shift estimation methods," in *Instrumentation and Measurement Technology Conference, 1996. IMTC-96. Conference Proceedings. Quality Measurements: The Indispensable Bridge between Theory and Reality., IEEE*, vol. 1. IEEE, 1996, pp. 162–167.

[3] X. Hu, V. Nenov, and M. Bergsneider, "Phase coordination between intracranial pressure and electrocardiogram signals," in *Engineering in Medicine and Biology Society, 2005. IEEE-EMBS 2005. 27th Annual International Conference of the*. IEEE, 2006, pp. 45–48.

[4] H.-S. Song and K. Nam, "Instantaneous phase-angle estimation algorithm under unbalanced voltage-sag conditions," *IEE Proceedings-Generation, Transmission and Distribution*, vol. 147, no. 6, pp. 409–415, 2000.

[5] N. Soorma, J. Singh, and M. Tiwari, "Feature extraction of ecg signal using hht algorithm," *arXiv preprint arXiv:1403.1660*, 2014.

[6] T. P. Zielinski, "Wigner transform instantaneous phase estimator," in *European Signal Processing Conference, 1996. EUSIPCO 1996. 8th*. IEEE, 1996, pp. 1–4.

[7] D. Benitez, P. Gaydecki, A. Zaidi, et al., "The use of the hilbert transform in ecg signal analysis," *Computers in biology and medicine*, vol. 31, no. 5, pp. 399–406, 2001.

[8] L. Rantner, L. Wieser, M. Stühlinger, et al., "Detection of phase singularities in triangular meshes," *Methods of information in medicine*, vol. 46, no. 6, pp. 646–654, 2007.

[9] P. Kuklik, S. Zeemering, B. Maesen, et al., "Reconstruction of instantaneous phase of unipolar atrial contact electrogram using a concept of sinusoidal recomposition and hilbert transform," *IEEE transactions on biomedical engineering*, vol. 62, no. 1, pp. 296–302, 2015.

[10] atan2. https://en.wikipedia.org/wiki/Atan2.

[11] A. Ingle, "The modified constant q spectrogram (mcqs) and its application to phase vocoding," PhD thesis, UNIVERSITY OF WISCONSIN-MADISON, 2011.

[12] M. Wilhelms, "Multiscale modeling of cardiac electrophysiology," PhD thesis, Karlsruhe Institute of Technology, 2013.

[13] M. K. Stiles, A. G. Brooks, B. John, et al., "The effect of electrogram duration on quantification of complex fractionated atrial electrograms and dominant frequency," *Journal of cardiovascular electrophysiology*, vol. 19, no. 3, pp. 252–258, 2008.

[14] Noise (signal processing). https://en.wikipedia.org/wiki/Noise_(signal_processing).

[15] Noise. https://en.wikipedia.org/wiki/Noise.

[16] White noise. https://www.probabilitycourse.com/chapter10/10_2_4_white_noise.php.

[17] Gaussian distribution function. http://www.gaussianwaves.com/2013/11/simulation-and-analysis-of-white-noise-in-matlab/.

[18] Hamming window. https://de.mathworks.com/help/signal/ref/hamming.html.

[19] Corelation coefficients. http://www.investopedia.com/terms/v/variable-cost-ratio.asp.

[20] Covariance. https://en.wikipedia.org/wiki/Covariance.

[21] Corelation coefficient. https://de.mathworks.com/help/matlab/ref/corrcoef.html.

[22] Standard deviation. https://en.wikipedia.org/wiki/Standard_deviation.