Inclusion of Parity Bits in Molecular Communication Systems

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***Abstract*—Molecular communication (MC) is a novel field with the potential for much substantial growth. MC enables communication among nanoscale devices previously impossible with conventional RF-based communication. However, MC systems must overcome high rates of inter-symbol interference (ISI) and Brownian noise. These factors cause MC channels to exhibit much higher bit error rates compared to standard wireless channels. In this paper, we propose the introduction of parity bits into a molecular communication system to reduce the bit error rate (BER) and increase the data rate. We simulate the impact of parity bits on the BER and data rate while varying the message size and properties of the MC system such as the distance between the transmitter and receiver and the radius of the receiver. We also simulate the impact of a 1-bit parity bit on the system. For both error detection schemes, we find a substantial improvement in the BER and data rate of the system.**

***Index Terms*—Molecular Communications, Computer Networking**

I. Introduction

In recent years, molecular communication (MC) has garnered considerable research attention, as it offers a novel approach to enabling communication among nanoscale devices [1]. As the name suggests, MC utilizes molecules to transmit information. Given their exceptionally small size, molecules enable MC to be utilized in a wide range of nanoscale applications where conventional RF-based communication faces limitations due to various constraints [2].

Nanomachines are miniature devices capable of facilitating communication. The basic model of an MC system model includes a transmitter nanomachine, a fluid medium, and a receiver nanomachine. Information-carrying molecules travel through the fluid medium through mechanisms such as diffusion or flow-assisted diffusion [3]. MC systems are particularly well-suited for integration with biological systems performing molecular-level tasks due to their inherent biocompatibility [4]. For example, in healthcare applications, nanomachines can be programmed to identify and neutralize pathogens within the human body.

Despite these benefits, MC systems have high bit error rates (BER) and low data rates, with channels that are characterized by slow propagation rates due to the diffusion mechanism and are susceptible to high levels of intersymbol interference (ISI) [5]. In this paper, we develop and model a simple MC system with parity bit-based error detection and simulate its impacts on the system BER and data rate. In Sec. II, we introduce the MC system model as well as the two noise sources present in the molecular channel, Brownian noise and Residual noise (ISI). In Sec. III, we look at related works that incorporate various error detection techniques in MC systems. In Sec. IV, we discuss the methodology used in our simulations. This includes both implementation of parity bits into the system model, error detection and packet retransmission, and simulation setup. In Sec. V, we present our simulated results and discuss our findings and how they compare to the existing works mentioned in Sec. III. In Sec. VI we conclude the paper and identify possible future research directions.

II. System Model

For an MC system, we consider two nanomachines within an unbounded three-dimensional space: a point transmitter (TN) at the origin, (0,0,0), and a passive receiver (RN) at point $(x_\mathrm{r}, y_\mathrm{r}, z_\mathrm{r})$ centered within a sphere of radius r as shown in Fig.1. At time $t = 0$, the TN begins emitting molecules that propagate through the channel. The molecules undergo Brownian motion as they move through the medium. The RN continuously counts the number of molecules received within sphere V_R at any time t. For this project, we assume that the transmitter and receiver are perfectly synchronized [6]. The MC system uses an On-Off Keying (OOK) modulation scheme for data transmission. In this scheme, the TN will emit Q molecules to transmit a binary ”1,” while the absence of molecule emission represents the transmission of a binary ”0”. The TN releases all Q molecules instantaneously at the start of the symbol period where a ”1” will be transmitted. We model this emission mathematically as an impulse represented by the following expression

$$s(t) = Q\delta(t), \tag{1}$$

where $Q \gg 1$ and $\delta(t)$ is the Dirac delta function.

Once emitted, the molecules will spread freely throughout the medium via diffusion. This diffusion process is governed by Fick’s law, which explains molecular movement based on the concentration gradient between transmitter and receiver. We let $c(x, y, z, t)$ represent the molecular concentration at a given location (x, y, z) at time t. The random motion of molecules during diffusion causes fluctuations in $c(x, y, z, t)$

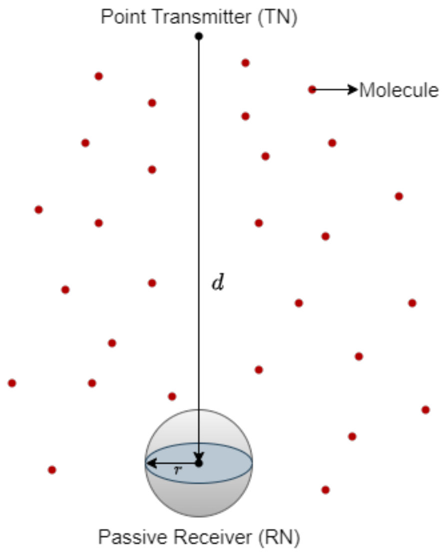

Fig. 1: System Model

over time and space, following Fick's second law of diffusion [7]

$$\frac{\partial c(x,y,z,t)}{\partial t} = D\nabla^2 c(x,y,z,t), \tag{2}$$

where D is the diffusion coefficient, and ∇^2 is the Laplacian operator of (x,y,z). The necessary initial conditions for determining the channel impulse response (CIR) are as follows:

$$c(x,y,z,t=0) = \delta(x,y,z). \tag{3}$$

The CIR, $h(t)$, can be derived using Equations (2) and (3) and is given by the following expression [8]:

$$h(t) = \frac{V_R}{(4\pi Dt)^{\frac{3}{2}}} \exp\left(-\frac{(d-vt)^2}{4Dt}\right), \tag{4}$$

where v represents the drift velocity within the channel and d denotes the distance between the TN and RN, calculated as $d = \sqrt{x_{\rm r}^2 + y_{\rm r}^2 + z_{\rm r}^2}$. The channel's response to the input signal $s(t)$ is given by $x(t)$, which is determined by convolving the input signal with the CIR, expressed as $x(t) = s(t) \circledast h(t)$. Physically, $x(t)$ represents the average number of molecules detected within the receiver's region, excluding noise effects. During the kth symbol period, the mean number of molecules in the reception region is given by

$$x(k,t) = a(k)Qh(t), \tag{5}$$

where $a(k)$ is the symbol transmitted in the kth period, either "1" or "0".

A. Brownian Noise

The first source of noise, known as Brownian noise and denoted by $n_b(k,t)$, is an additive, signal-dependent noise caused by the random thermal motion of messenger molecules. Let $w(k,t)$ represent the total number of molecules detected within the receiver's region, which can be described as:

$$w(k,t) = x(k,t) + n_b(k,t). \tag{6}$$

To analyze Brownian noise, we define two quantities: the noiseless and noisy molecule count, $N(k,t)$, $\hat{N}(k,t)$, in the reception region. These are given by:

$$N(k,t) = x(k,t); \quad \hat{N}(k,t) = w(k,t). \tag{7}$$

The number of molecules, b, detected in the RN's reception region follows a Binomial distribution [9], expressed as

$$p(b) = \binom{Q}{b} p^b (1-p)^{Q-b}. \tag{8}$$

Here p is the probability that a molecule will reach the reception region. For $Q \gg 1$, this Binomial distribution can be approximated as a Gaussian distribution $\mathcal{N}(\mu, \sigma^2)$, where:

$$\mu = Qp, \quad \sigma^2 = Qp(1-p). \tag{9}$$

Here, μ represents the expected (noiseless) number of molecules detected by the RN, given by $\mu = Qp = N(k,t)$. Assuming $1-p \simeq 1$, the variance becomes $\sigma^2 = Qp(1-p) = Qp = N(k,t)$. Consequently, the number of received molecules $\hat{N}(k,t)$ follows a Gaussian distribution $\hat{N}(k,t) \sim \mathcal{N}(N(k,t), N(k,t))$, where $\mathcal{N}(x,y)$ denotes a normal distribution with mean x and variance y. Using Equation (7), the received noisy molecule count, $w(k,t)$, is distributed as $w(k,t) \sim \mathcal{N}(x(k,t), x(k,t))$. Thus, the Brownian noise, $n_b(k,t)$, follows a Gaussian distribution with zero mean and variance equal to $x(k,t)$), expressed as $n_b(k,t) \sim \mathcal{N}(0, x(k,t))$. Finally, the variance of the Brownian noise is given by

$$\sigma_b^2 = a(k)Qh(t), \tag{10}$$

B. Residual Noise

In scenarios with multiple symbol periods, molecules emitted during earlier periods can remain in the channel. These residual molecules contribute to a secondary type of noise known as residual noise, represented as $n_r(k,t)$. This noise can also be described as Inter-Symbol Interference (ISI) and can be approximated by summing the contributions from the tails of all preceding symbols. As shown in Equation (9), each individual tail exhibits a Gaussian distribution. Therefore, the total number of residual molecules is also Gaussian. Therefore, $n_r(k,t)$ can be expressed as [10],

$$n_r(k,t) \sim \mathcal{N}(\mu_r, \sigma_r^2), \tag{11}$$

where for the kth symbol period, we have

$$\mu_r = \sum_{j=1}^{k-1} a(k-j)Qh(t_s + jT), \tag{12}$$

$$\sigma_r^2 = \sum_{j=1}^{k-1} a(k-j)Qh(t_s + jT)(1 - h(t_s + jT)), \tag{13}$$

where T is the symbol duration.

The overall performance of the communication link is influenced by both Brownian noise and residual noise. For each symbol period, the RN measures the number of molecules at a time t_s from the beginning of the symbol period. Consequently, during the kth symbol, at t_s time, the variances σ_b^2 and σ_r^2 can be expressed as follows:

$$\sigma_b^2 = a(k)Qh(t_s), \tag{14}$$

$$\sigma_r^2 = \sum_{j=1}^{k-1} a(k-j) \times Q \times h(t_s + jT) \times (1 - h(t_s + jT)). \tag{15}$$

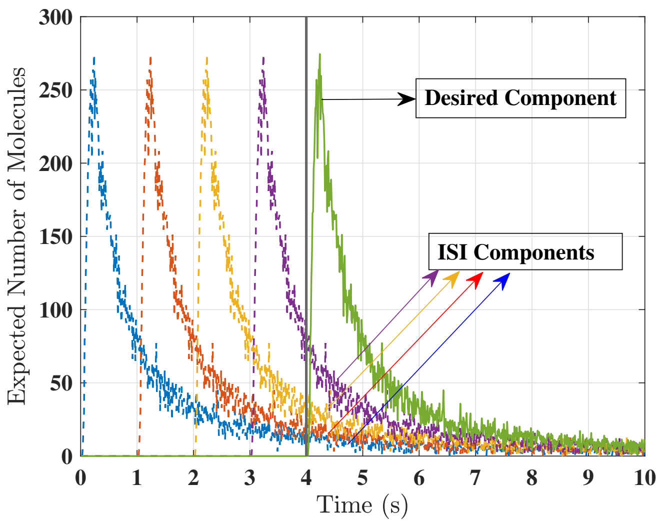

Fig. 2: ISI model in molecular communication systems

C. Expected Number of Molecules at the RN

Let $y(k,t)$ represent the number of molecules observed by the RN after accounting for both noises, which is described as:

$$y(k,t) = x(k,t) + n_b(k,t) + n_r(k,t). \tag{16}$$

The expected number of molecules at the RN after accounting for Brownian noise and ISI is shown in Fig. 2. In Fig. 2, the duration of a symbol period is 1 second. For each symbol transmitted by the TN, the previous symbols will act as ISI, represented by the dotted lines.

III. Background and Related Work

Incorporating error detection and correction in MC systems is a new but growing field. One focus area related to our work is the development of new detection schemes for MC systems as well as the adaptation of preexisting detection schemes to MC [5], [11]–[14]. A key distinction between MC channels and wireless communication channels is the ISI caused by previous transmitted symbols. Standard MC systems assume a fixed threshold, ζ_t, such that a "1" is received if number of molecules at the RN is greater than ζ_t [15]. However, the molecules at the RN can be inflated by the ISI caused by remaining molecules from earlier transmissions. Adaptive threshold detection is one such method to mitigate the impact of the increased ISI found in the MC channel. Under adaptive threshold detection, the receiver retains memory of previous received bits and adjusts its threshold accordingly [13], [14].

As we focus on incorporating parity bit based error detection, the adaptation of preexisting error detection schemes is more relevant to this paper. One such work has focused on adapting Hamming distances to better fit the conditions of MC channels, proposing a new distance function that better accounts for the Brownian noise present in MC [11]. The feasibility of single-parity check codes in a MC biological circuit is shown by [12]. Their model demonstrated that an analog biological circuit could use log-likelihood ratios to achieve a similar BER to a comparable electrical implementation. We hope to confirm this feasibility in our paper and expand upon its findings to a generic MC system.

IV. Methodology

This section discusses the methodology used in this project. It focuses on how we incorporated parity bits into the system model introduced in Sec. II, the various variables we consider in our estimations of BER and data rate, and the way the results are plotted and relevant data extracted out of them.

A. Characterizing the Physical Channel

The physical channel is the medium through which communication occurs. There exist several molecule propagation models such as free diffusion, flow-based propagation, and bacterial-assisted propagation. Among these, free diffusion model is the most common and widely studied [3]. As discussed in Sec. II, we can model this diffusion using Equation (2). To model the number of molecules at the receiver, we must consider the physical channel, the Brownian noise, and the ISI as given by Equation (16). To demonstrate the feasibility of parity bits for MC systems, the system must be capable of transmitting over multiple symbol periods. Each consecutive bit will continue to add to the ISI of the system for the next symbol period. An overlay of the symbol periods for an 8 bit packet can be seen in Figure 3. Despite the fact that the transmitter does not emit molecules when transmitted a "0", the ISI still causes the receiver to detect some molecules. We set the threshold to be the mean number of molecules at the receiver halfway through the symbol period at $T/2$. We sample each bit halfway through the symbol period at sample 50. To get a sense for how the MC channel impacts the BER and data rate, we conduct several simulations varying the radius of the receiver's detection area and the distance between the transmitter and receiver.

B. Adding Parity Bits

For this project, we consider packet sizes of 4, 8, 12, and 16 bits, with the last bit of each being a parity bit. The data bits are randomly generated, and the parity bit calculation is then given by:

$$p_b = \left(\bigoplus_{i=1}^{N} d_i \right) \mod 2 \tag{17}$$

where N is the total number of data bits and d_i is the current bit. The packet is then transmitted one bit at a time, through the process shown in Fig. 3. Once the packet arrives at the receiver, the expected parity bit, p_E is calculated using Equation (17). The expected and received parity bits are then compared using

$$p_r = (p_b \oplus p_E) \mod 2 \quad (18)$$

If the parity bits are not equal, the receiver flags the packet and it is resent by the transmitter. This assumes that there is some way for the receiver to notify the transmitter to resend the packet. The receiver has no way of detecting errors if there are an even number of bit flips, so those errors will still get through.

C. Error Detection and Calculations

For a given packet of size N, we can calculate the BER using the equation:

$$BER = E/N \quad (19)$$

where E is the total number of erroneous bits and N is the total number of bits transmitted. The data rate is then calculated by:

$$R = \frac{d_N}{TN} \quad (20)$$

where d_N is the total number of data bits sent, T is the symbol period and N is the total number of transmitted bits. In our system model, T = 1.

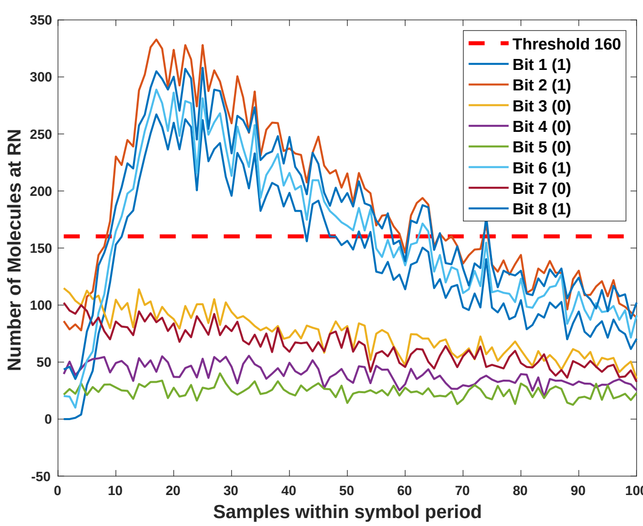

Fig. 3: Overlay of symbol periods for 8 bit packet in MC system

V. Results and Discussion

This section provides our simulation results, focusing on the impacts of parity bits on both the BER and data rate. We also consider the impacts of the physical channel properties, such as the distance between the transmitter and receiver, and the radius of the receiver. We assess any potential contributions and trade-offs in BER and data rate given these channel properties. All results were obtained using Monte Carlo simulations. Figure 4 shows the average BER and data rate for both parity and non-parity systems conducted over 1,000 simulated packet transmissions. For these simulations, the receiver had a radius of 2 μm and the receiver distance was 12 μm. We see also in Figure 5 that as the distance increases, the BER increases. As the distance to the receiver increases, the likelihood that a molecule approaches RN decreases proportionally. This leads to higher ISI as there are more remaining molecules in the channel. We see in Figure 6 that the BER decreases for larger radii. For larger radii, the receiver can detect a higher number of data molecules relative to residual ones, reducing the impact of ISI.

Message size (bits)	BER without parity bit	BER with parity bit	Decrease in BER with parity
M = 4	0.2008	0.0042	97.9%
M = 8	0.1268	0.0061	95.17%
M = 12	0.0887	0.0048	94.6%
M = 16	0.0654	0.0044	93.3%

Message size (bits)	Data rate (bps) without parity	Data rate (bps) with parity	Increase in data rate with parity
M = 4	1.63	1.69	3.5%
M = 8	2.71	3.65	26%
M = 12	3.94	5.64	30%
M = 16	5.42	7.85	31%

Fig. 4: Simulated impact of parity bit error correction on BER and data rate for a molecular communication system

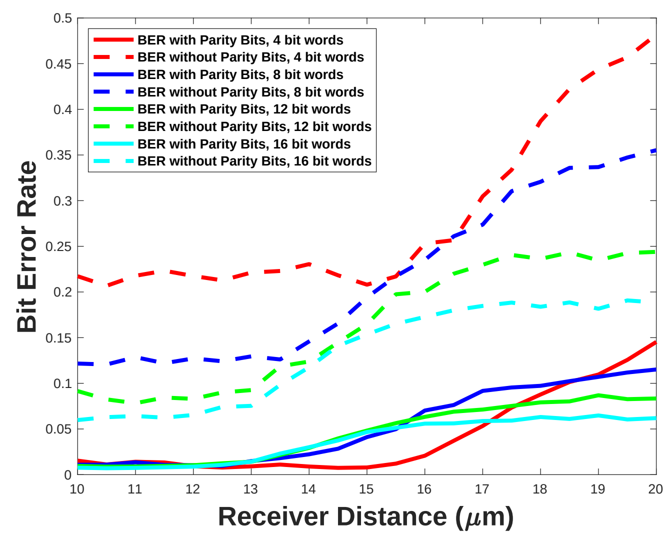

Fig. 5: BER across varying channel distances for parity and non-parity systems.

A. Parity Bit Impacts on BER

The changes in BER caused by the inclusion of parity bits can be seen in Figures 4, 5, and 6. Over 1,000 transmitted packets, we observe that the addition of a single parity bit to the end of every transmitted packet and single-bit error correction reduces the BER substantially. We find this to be the case regardless of the packet size or physical channel properties. As shown in Figure 4, the inclusion of parity bits reduced the BER by more than 90% for all packet sizes.

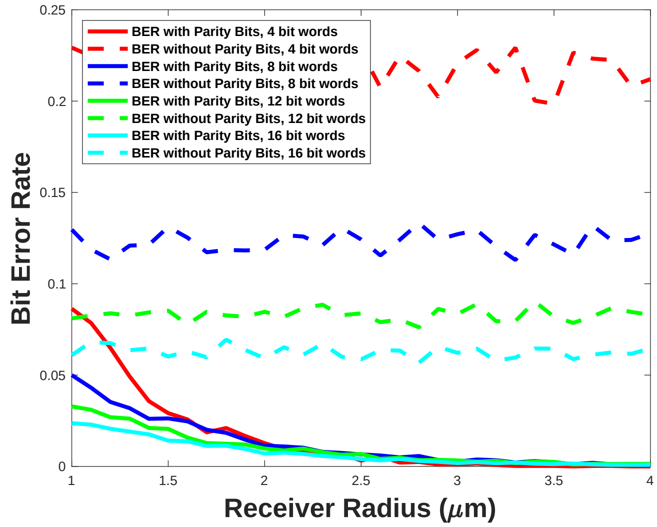

Fig. 6: BER across varying receiver radii for parity and non-parity systems.

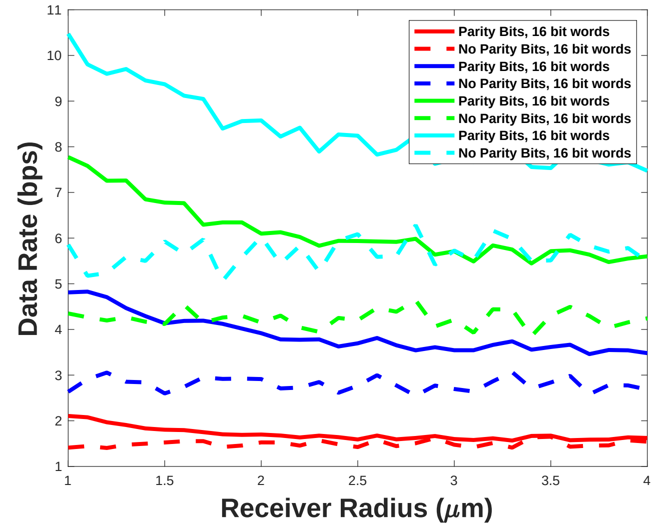

Fig. 8: Data rate across varying receiver radii for parity and non-parity systems.

However, the relative impact of adding parity bits on BER decreases slightly as the message size increases. This is caused by the higher likelihood of an even number of corrupted bits, since the parity bit can only detect single-bit errors in the codeword. This could be mitigated by using a more advanced error correction scheme.

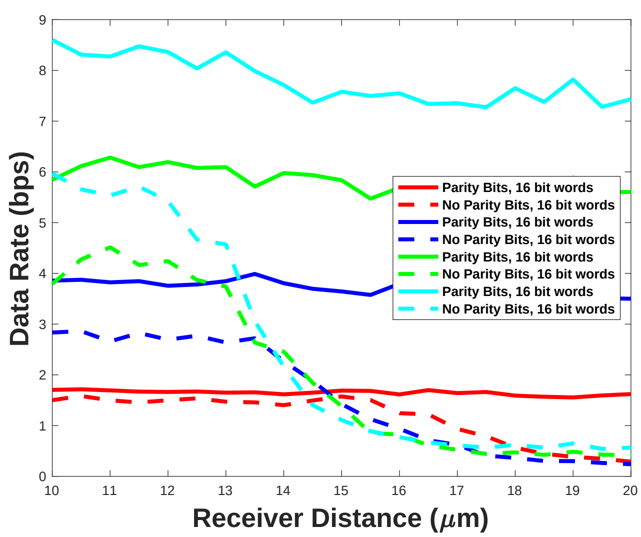

Fig. 7: Data rate across varying channel distances for parity and non-parity systems.

B. Parity Bit Impacts on System Data Rate

The inclusion of parity bits also has a positive impact on the data rate. As shown in Figures 4, 7 and 8, we find that the implementation of parity bits in the MC system increases the data rate by 25-30% for messages longer than 4 bits. The data rate of the MC systems with parity also retained higher rates at longer distances, while the non-parity systems experienced a significant decline across all word sizes.

VI. Conclusion

In this paper, we studied the impact of parity bits in a molecular communication (MC) system. We considered a basic MC system with a point transmitter and a passive spherical receiver. In addition to normal data bits, the transmitter sends parity bits to improve the system's performance. We investigated the impact of the radius of the receiver and the distance between the transmitter and receiver on the bit error rate (BER) and data rate of the system. The results showed that by including parity bits in the transmission process, the BER reduces. Furthermore, the data rate is improved relative to an MC system without parity bits. The proposed algorithm to include parity bits during the transmission process in MC systems increases the resiliency and makes the system more robust and is helpful in scenarios where the security of the system is a major concern.

VII. Team Contributions

Akarsh developed the system model for the MC channel and wrote Sections I and II. Rittwik developed the algorithm for parity bit implementation. Sebastien developed the algorithm for calculating the BER and data rate and wrote Sections III, IV, and V. All three team members contributed equally to the poster

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